

3. feladatsor – Halmazok

3.1. Feladat megoldása.

- | | | | |
|----------|----------|-----------|----------|
| (a) Igaz | (c) Igaz | (e) Hamis | (g) Igaz |
| (b) Igaz | (d) Igaz | (f) Igaz | (h) Igaz |

3.2. Feladat megoldása.

- $A \cup B = \{a, b, c, d, e\} = U$;
- $A \cap B = \{d\}$;
- $\overline{B} = \{a, b, c\}$;
- $A \setminus B = \{a, b, c\}$;
- $A \Delta B = \{a, b, c, e\}$;
- $(A \Delta \overline{C}) \setminus \overline{B} = \emptyset$;
- $\mathcal{P}(B) = \{\emptyset, \{d\}, \{e\}, \{d, e\}\}$.

3.3. Feladat megoldása.

$$A = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}, \quad B = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$$

- $A \cup B = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$
- $A \cap B = \{\emptyset, \{b\}\}$
- $A \setminus B = \{\{a\}, \{a, b\}\}$
- $B \setminus A = \{\{c\}, \{b, c\}\}$
- $A \Delta B = \{\{a\}, \{c\}, \{a, b\}, \{b, c\}\}$

3.4. Feladat megoldása. $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}$

3.5. Feladat megoldása.

- | | |
|----------|---------|
| (a) Igen | (d) Nem |
| (b) Igen | (e) Nem |
| (c) Nem | (f) Nem |

3.6. Feladat megoldása.

- (a) $\mathcal{C}_1 = \{\{1, 2\}, \{3, 4\}, \{5, 6\}, \{7\}\}$
- (b) $\mathcal{C}_2 = \{\{1, 2\}, \{3, 4\}, \{5, 6, 7\}\}$
- (c) $\mathcal{C}_3 = \{\{1, 2, 3\}, \{4, 5, 6, 7\}\}$
- (d) Nincs ilyen osztályozás.

3.7. Feladat megoldása.

- | | |
|--|--|
| (a) $(A \setminus B) \setminus B = A \setminus B$ | (e) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ |
| (b) $A = (A \cup B) \setminus (B \setminus A)$ | (f) $(A \cap B) \setminus (B \setminus (A \cup C)) = A \cap B$ |
| (c) $A \setminus (B \setminus C) \neq (A \setminus B) \setminus C$ | (g) $(A \Delta B) \Delta (A \cap B) = A \cup B$ |
| (d) $A \cap (B \cup C) \neq (A \cup B) \cap (A \cup C)$ | |

3.8. Feladat megoldása. $\overline{A \cup (B \cap (C \cup D))} = \overline{A} \cap (\overline{B} \cup (\overline{C} \cap \overline{D}))$

3.9. Feladat megoldása. Van: $A = \emptyset, B = \{\emptyset\}, C = \{\emptyset, \{\emptyset\}\}$.

3.10. Feladat megoldása. $A \subseteq B$ teljesül, $A = B$ és $B \setminus A = \emptyset$ nem teljesül.

3.11. Feladat megoldása.

(a)

$$\begin{aligned} x \in (A \cap C) \cup (B \cap D) &\iff x \in (A \cup (B \cap D)) \cap (C \cup (B \cap D)) \\ &\iff x \in (A \cup B) \cap (A \cup D) \cap (C \cup B) \cap (C \cup D) \\ &\implies x \in (A \cup B) \cap (C \cup D) \end{aligned}$$

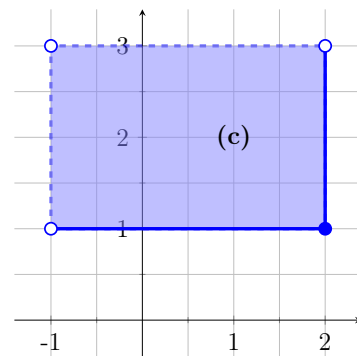
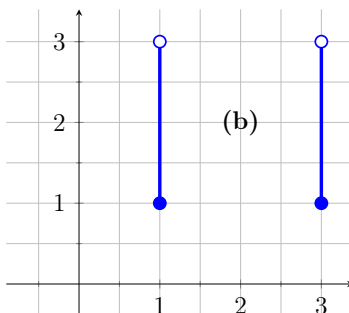
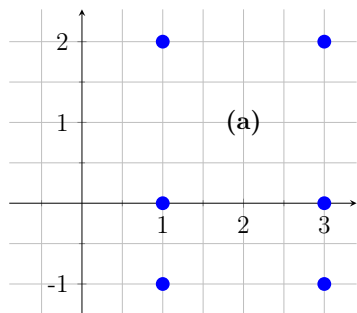
(b)

$$\begin{aligned} x \in C \cup D &\implies x \notin B \cap \overline{(C \cup D)} = B \setminus (C \cup D) \\ &\nearrow \\ x \in A \cap C \cap D & \\ &\searrow \\ &x \in A \cap C \\ &\implies x \in (A \cap C) \setminus (B \setminus (C \cup D)) \end{aligned}$$

3.12. Feladat megoldása.

- $A \cap A = \overline{A \cap A} = \overline{A}$
- $(A \cap B) \cap (A \cap B) = \overline{A \cap B} = \overline{\overline{A \cap B}} = A \cap B$
- $A \cup B = (A \cap A) \cap (B \cap B)$

3.13. Feladat megoldása.



3.14. Feladat megoldása.

Igen

Igen

Nem

Nem