

6. feladatsor – Komplex számok, polinomok

6.1. Feladat megoldása.

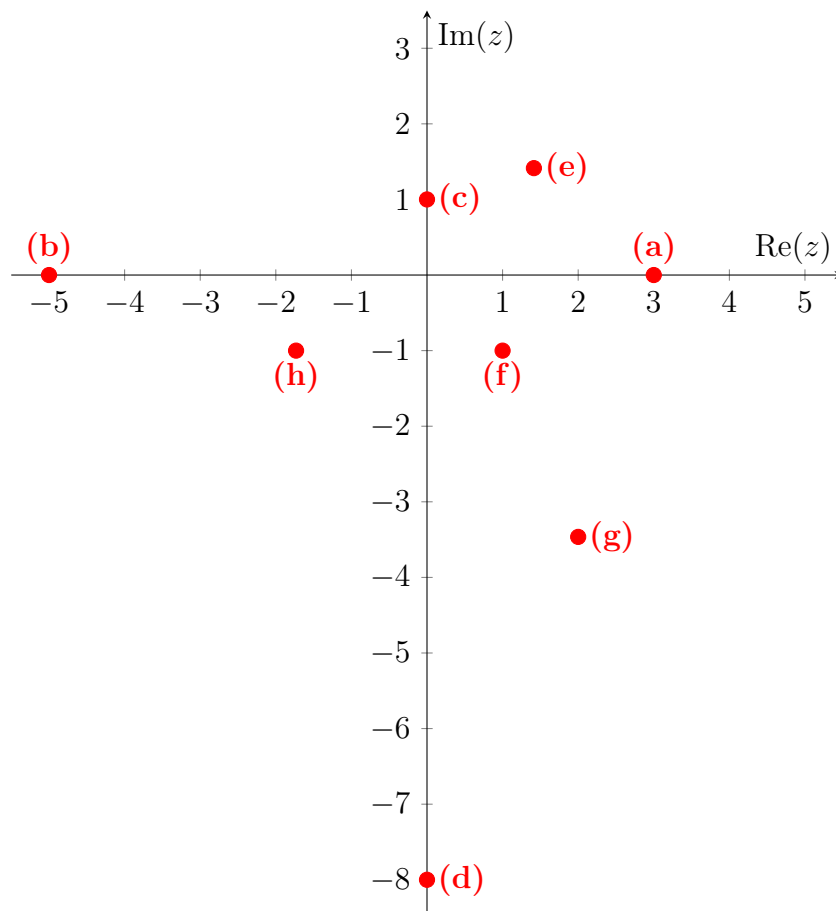
- (a) $-i$; -1 ; (b) $41 - 11i$; (c) $17 - 2i$; (d) $-\frac{11}{17} + \frac{27}{17}i$; (e) $-\frac{3}{13} - \frac{11}{13}i$; (f) $\frac{11}{10} - \frac{23}{10}i$;
(g) $-\frac{2}{5} + \frac{3}{10}i$.

6.2. Feladat megoldása.

- (a) $z = -\frac{13}{10} + \frac{11}{10}i$
(b) $z = 3 - i$
(c) $z_1 = 2 + i$, $z_2 = 2 - i$, $z_3 = \frac{3}{2}$
(d) $z_1 = 0$, $z_2 = -i$, $z_3 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$, $z_4 = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$.

6.3. Feladat megoldása.

- (a) $3 = 3 \cdot (\cos 0 + i \cdot \sin 0) = 3e^0$; (e) $\sqrt{2} + \sqrt{2}i = 2(\cos \frac{\pi}{4} + i \cdot \sin \frac{\pi}{4}) = 2e^{\frac{\pi}{4}i}$;
(b) $-5 = 5 \cdot (\cos \pi + i \cdot \sin \pi) = 5e^{\pi i}$; (f) $1 - i = \sqrt{2}(\cos \frac{7\pi}{4} + i \cdot \sin \frac{7\pi}{4}) = \sqrt{2}e^{\frac{7\pi}{4}i}$;
(c) $i = \cos \frac{\pi}{2} + i \cdot \sin \frac{\pi}{2} = e^{\frac{\pi}{2}i}$; (g) $2 - 2\sqrt{3}i = 4(\cos \frac{5\pi}{3} - i \cdot \sin \frac{5\pi}{3}) = 4e^{\frac{5\pi}{3}i}$;
(d) $-8i = 8(\cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2}) = 8e^{\frac{3\pi}{2}i}$; (h) $-\sqrt{3} - i = 2(\cos \frac{7\pi}{6} - i \cdot \sin \frac{7\pi}{6}) = 2e^{\frac{7\pi}{6}i}$.



6.4. Feladat megoldása.

(a) $2(\cos 0 + i \sin 0) = 2;$

(b) $3e^{\frac{3\pi}{2}i} = -3i;$

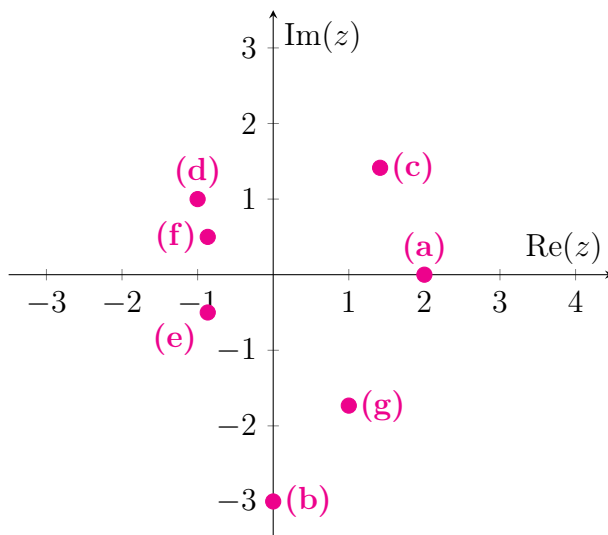
(c) $2e^{\frac{\pi}{4}i} = \sqrt{2} + \sqrt{2}i;$

(d) $\sqrt{2}(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}) = -1 + i;$

(e) $\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i;$

(f) $e^{\frac{5\pi}{6}i} = -\frac{\sqrt{3}}{2} + \frac{1}{2}i;$

(g) $2(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}) = 1 - \sqrt{3}i.$



6.5. Feladat megoldása.

(a) $8(\cos \frac{\pi}{6} + i \cdot \sin \frac{\pi}{6}) = 4\sqrt{3} + 4i;$

(b) $\cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2} = -i;$

(c) $\cos(-\frac{\pi}{6}) + i \cdot \sin(-\frac{\pi}{6}) = \frac{\sqrt{3}}{2} - \frac{1}{2}i;$

(d) $\cos \pi + i \cdot \sin \pi = -1;$

(e) $2^{67}(\cos \frac{5\pi}{6} + i \cdot \sin \frac{5\pi}{6});$

(f) $2^{611}(\cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2});$

(g) $6^{1526}(\cos \frac{2\pi}{3} + i \cdot \sin \frac{2\pi}{3}).$

6.6. Feladat megoldása.

(a) $2i = 2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$
 $-2i = 2(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2})$

(b) $-2 = 2(\cos \pi + i \sin \pi)$
 $1 + \sqrt{3}i = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$
 $1 - \sqrt{3}i = 2(\cos(-\frac{\pi}{3}) + i \sin(-\frac{\pi}{3}))$

(c) $\cos \frac{\pi}{8} + i \sin \frac{\pi}{8},$
 $\cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8},$
 $\cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8},$
 $\cos \frac{13\pi}{8} + i \sin \frac{13\pi}{8}.$

(d) $2 = 2(\cos 0 + i \sin 0),$
 $1 + \sqrt{3}i = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}),$
 $-1 + \sqrt{3}i = 2(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}),$

$-2 = 2(\cos \pi + i \sin \pi),$
 $-1 - \sqrt{3}i = 2(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}),$
 $1 - \sqrt{3}i = 2(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}).$

(e) $2i = 2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}),$
 $-\sqrt{3} - i = 2(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}),$
 $\sqrt{3} - i = 2(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}).$

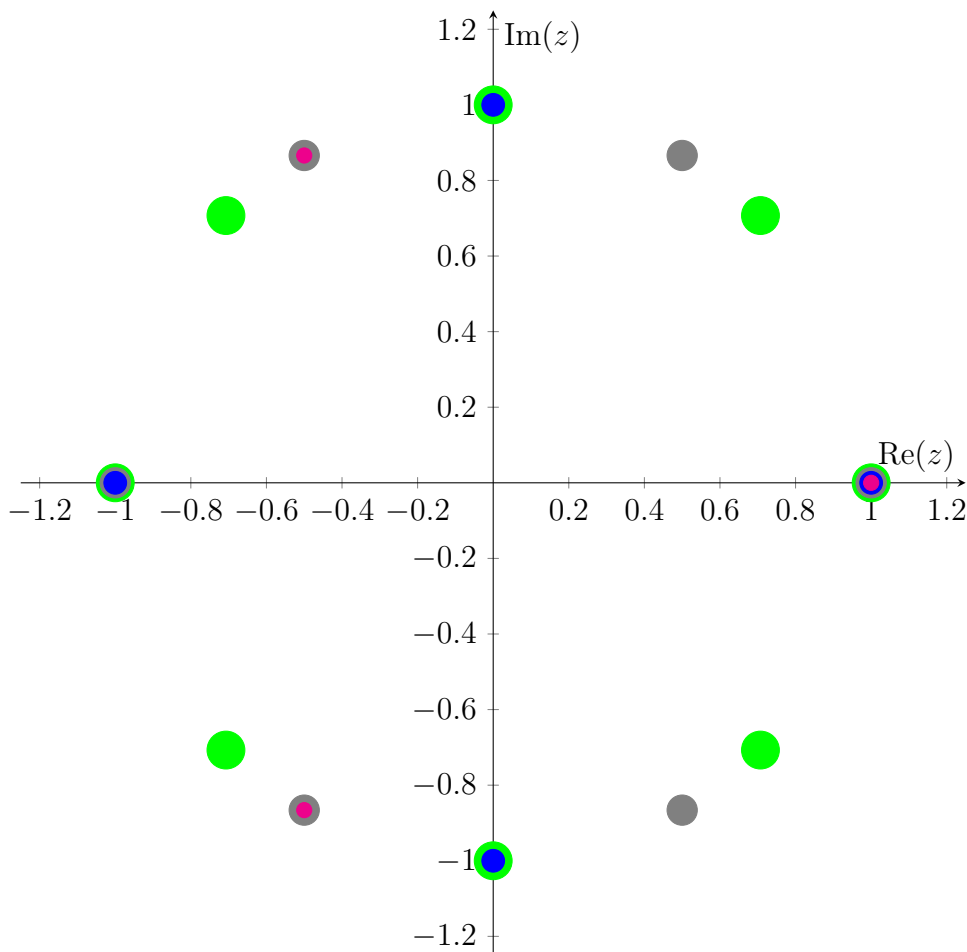
(f) $\sqrt[4]{2}(\frac{1}{2} + \frac{\sqrt{3}}{2}i) = \sqrt[4]{2}(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}),$
 $\sqrt[4]{2}(-\frac{\sqrt{3}}{2} + \frac{1}{2}i) = \sqrt[4]{2}(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}),$
 $\sqrt[4]{2}(-\frac{1}{2} - \frac{\sqrt{3}}{2}i) = \sqrt[4]{2}(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}),$
 $\sqrt[4]{2}(\frac{\sqrt{3}}{2} - \frac{1}{2}i) = \sqrt[4]{2}(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}).$

6.7. Feladat megoldása.

(a) Harmadik egységgyökök: $1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i.$

Primitív harmadik egységgyökök: $-\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i.$

- (b) **Negyedik** egységgyökök: $1, i, -1, -i$.
 Primitív negyedik egységgyökök: $i, -i$.
- (c) **Hatodik** egységgyökök: $1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -1, -\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$.
 Primitív hatodik egységgyökök: $\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} + \frac{\sqrt{3}}{2}i$.
- (d) **Nyolcadik** egységgyökök: $1, \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, i, -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -1, -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, -i, \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$.
 Primitív nyolcadik egységgyökök: $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$.



6.8. Feladat megoldása.

- (a) $x_1 = -3 - i, x_2 = -3 + i$
 $x^2 + 6x + 10 = (x + 3 + i)(x + 3 - i)$
- (b) $x_1 = 2i, x_2 = -\sqrt{3} - i, x_3 = \sqrt{3} - i$
 $2x^3 + 16i = (x - 2i)(x + \sqrt{3} + i)(x - \sqrt{3} + i)$
- (c) $x_1 = -2, x_2 = 1 + \sqrt{3}i, x_3 = 1 - \sqrt{3}i$
 $x^3 + 8 = (x + 2)(x - 1 - \sqrt{3}i)(x - 1 + \sqrt{3}i)$
- (d) $x_1 = \sqrt[4]{2} \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = \sqrt[4]{2} \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right),$
 $x_2 = \sqrt[4]{2} \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = \sqrt[4]{2} \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right),$
 $x_3 = \sqrt[4]{2} \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) = \sqrt[4]{2} \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right),$

$$x_4 = \sqrt[4]{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i \right) = \sqrt[4]{2} \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right).$$
$$x^4 + 1 + \sqrt{3}i = (x - x_1)(x - x_2)(x - x_3)(x - x_4)$$

6.9. Feladat megoldása.

(a) x^2

(b) $-x^2 + 8x - 4$

(c) $-\frac{13}{15}x^3 + 3x^2 + \frac{28}{15}x$