4. feladatsor – Relációk

4.1. Feladat megoldása.

$\alpha^{-1} = \{(2,1), (2,3), (4,3), (4,4), (5,5)\}$
$\alpha\beta = \{(1,2), (3,2), (3,3), (3,5), (4,3), (4,5)\}$
$\beta\alpha = \{(4,2), (4,4), (4,5)\}$
$\beta\alpha^{-1} = \{(1,1), (1,3), (2,1), (2,3), (3,1), (3,3), (4,5)\}$
$\alpha\beta^{-1} = \{(1,1), (1,2), (1,3), (3,1), (3,2), (3,3), (5,4)\}$

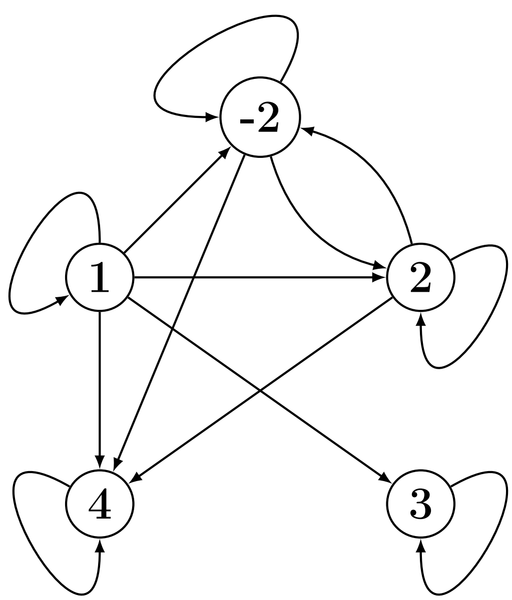
4.2. Feladat megoldása.

(a) $\alpha^{-1} = \{(y,x) \in \mathbb{E}^2 : \ x \text{ az } y \text{ gyermeke}\}$
$\alpha\beta = \{(x,y) \in \mathbb{E}^2 : \ y \text{ az } x \text{ nagyapja}\}$
$\beta\alpha = \{(x,y) \in \mathbb{E}^2 : \ y \text{ az } x \text{ apai nagyszülője}\}$

(b) $\alpha^{-1} = \{(y,x) \in \mathbb{R}^2 : \ x = 2y\}$
$\alpha\beta = \{(x,y) \in \mathbb{R}^2 : \ x = 2^{y+1}\}$
$\beta\alpha = \{(x,y) \in \mathbb{R}^2 : \ x = 4^y\}$

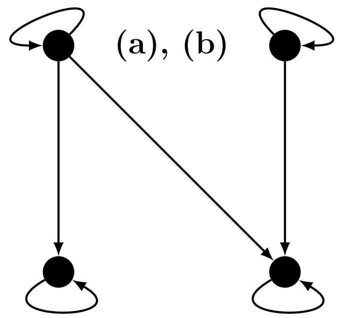
(c) $\alpha^{-1} = \{(y,x) \in \mathbb{R}^2 : \ y = x^2\}$
$\alpha\beta = \left\{(x,y) \in \mathbb{R}^2 : \ x^2 = \frac{y-1}{3}\right\}$
$\beta\alpha = \left\{(x,y) \in \mathbb{R}^2 : \ (3x+1)^2 = y\right\}$

4.3. Feladat megoldása.



Reflexív, nem szimmetrikus, nem antiszimmetrikus, tranzitív, nem dichotom.

4.4. Feladat megoldása.



(c): Nincs ilyen gráf, minden dichotom reláció reflexív.

4.5. Feladat megoldása.

	Refl.	Szimm.	Antiszimm.	Tranz.	Dich.	Ekv.	R.r.
(a)	✓	✓	×	×	×	×	×
(b)	✓	×	✓	✓	✓	×	✓
(c)	×	×	✓	✓	×	×	×
(d)	✓	✓	×	×	×	×	×
(e)	✓	×	×	✓	✓	×	×
(f)	✓	✓	×	✓	×	✓	×
(g)	✓	✓	×	✓	×	✓	×
(h)	✓	✓	×	✓	×	✓	×
(i)	×	×	✓	✓	×	×	×
(j)	✓	✓	×	✓	×	✓	×

4.6. Feladat megoldása.

$\mathcal{C} = \{\{1,2\},\{3,5\},\{4\}\}$

$\varrho = \{(1,1),(1,2),(2,1),(2,2),(3,3),(3,5),(5,3),(5,5),(4,4)\}$

4.7. Feladat megoldása.

(a) $\{\{-3,-2,-1\},\ \{1,2,3\}\}$

(b) $\{\{-3,0,3\},\ \{-2,1\},\ \{-1,2\}\}$

(c) $\{\{\emptyset\},\ \{\{\emptyset\},\{0\}\},\ \{\{1,2\},\{a,b\}\},\ \{\{1,2,3\}\}\}$

(d) $\{\{2,8,14,26\},\ \{3,9,15\},\ \{19\}\}$

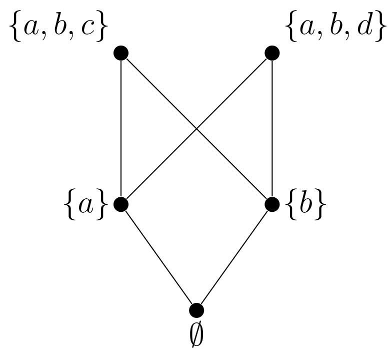
(e) $\{\{71,602\},\ \{301,4,121\},\ \{216,54,315\}\}$

(f) $\{\{0\},\ \{-1,1\},\ \{-2,2\},\ \{-3,3\},\ \ldots\} = \{\{a,-a\}\colon a \in \mathbb{Z}\}$

(g) $\{\{\ldots,-6,-4,-2,0,2,4,6,\ldots\},\ \{\ldots,-3,-1,1,3,\ldots\}\}$

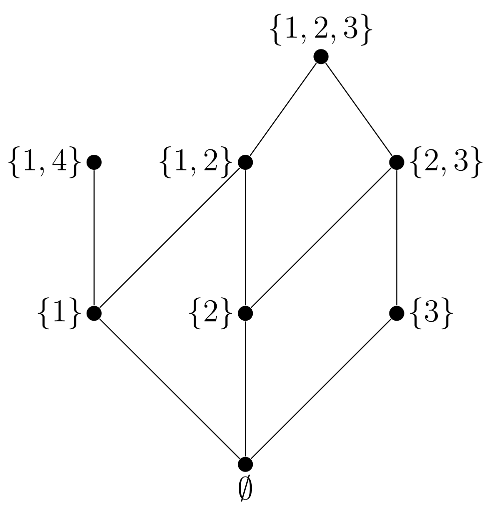
4.8. Feladat megoldása.

(a)



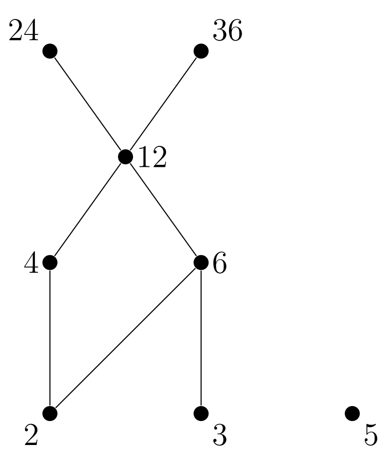
Legnagyobb elem: nincs.
Legkisebb elem: $\emptyset$.
Maximális elemek: $\{a,b,c\}$, $\{a,b,d\}$.
Minimális elem: $\emptyset$.

(b)



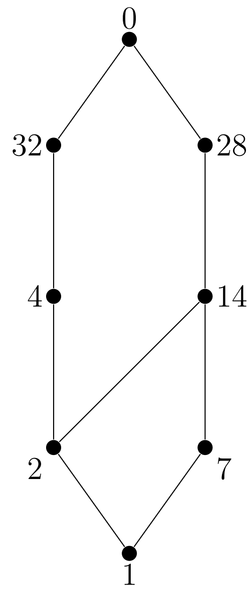
Legnagyobb elem: nincs.
Legkisebb elem: $\emptyset$.
Maximális elemek: $\{1,2,3\}$, $\{1,4\}$.
Minimális elem: $\emptyset$.

(c)



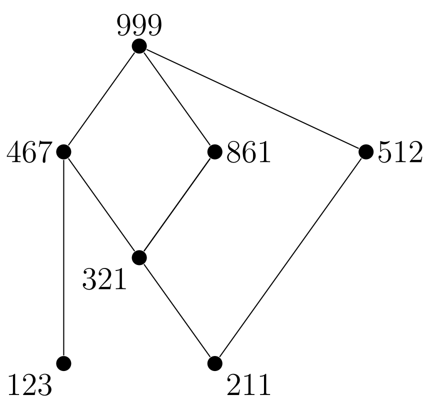
Legnagyobb elem: nincs.
Legkisebb elem: nincs.
Maximális elemek: 24, 36, 5.
Minimális elemek: 2, 3, 5.

(d)



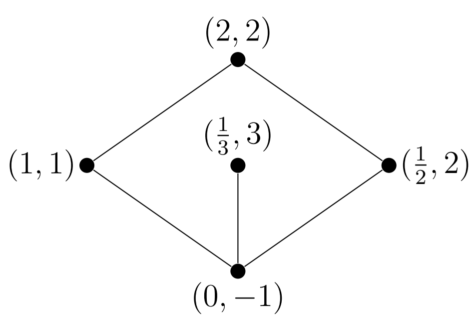
Legnagyobb elem: 0.
Legkisebb elem: 1.
Maximális elem: 0.
Minimális elem: 1.

(e)



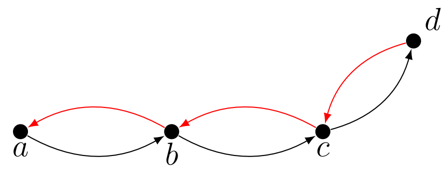
Legnagyobb elem: 999.
Legkisebb elem: nincs.
Maximális elem: 999.
Minimális elemek: 123, 211.

(f)

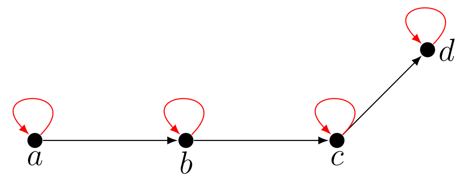


Legnagyobb elem: nincs.
Legkisebb elem: $(0,-1)$.
Maximális elemek: $(2,2)$, $(\frac{1}{3},3)$.
Minimális elem: $(0,-1)$.

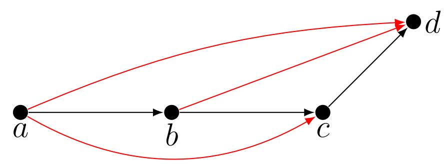
4.9. Feladat megoldása.
Szimmetrikus lezárt:



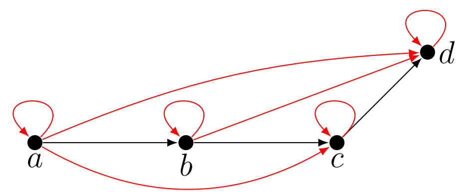
Reflexív lezárt:



Tranzitív lezárt:

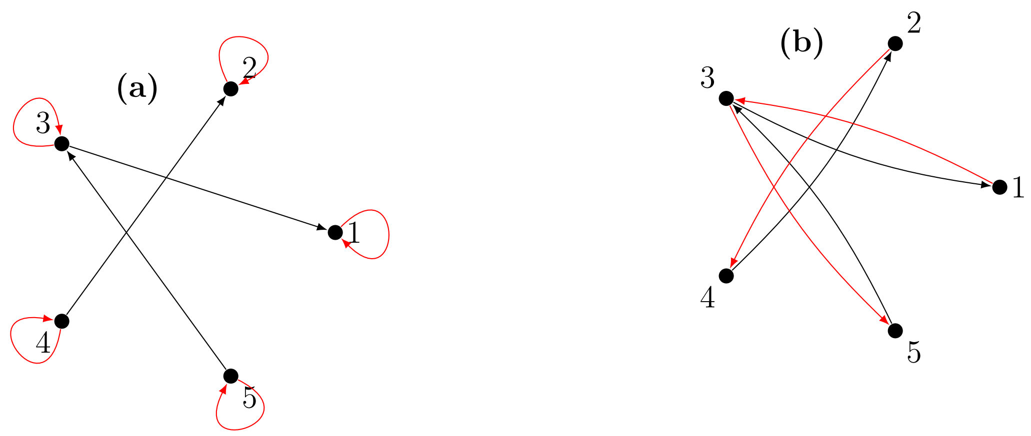


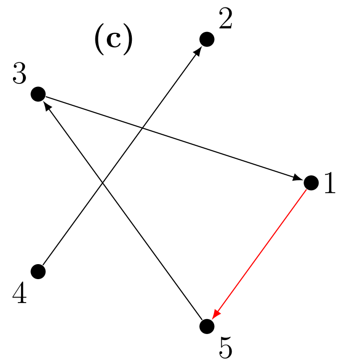
Reflexív és tranzitív lezárt:

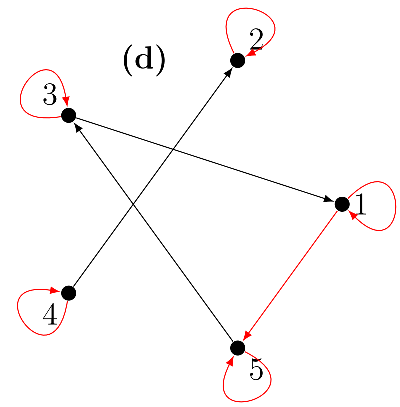


4.10. Feladat megoldása.

(a) ρ reflexív lezártja: $\{(4,2),(3,1),(5,3),(1,1),(2,2),(3,3),(4,4),(5,5)\}$
(b) ρ szimmetrikus lezártja: $\{(a,b)\colon |a-b|=2\}$
(c) ρ tranzitív lezártját: $\rho^+ = \rho \cup \{(1,5)\}$
(d) ρ reflexív és tranzitív lezártja: $\rho^* = \{(4,2),(3,1),(5,3),(1,1),(2,2),(3,3),(4,4),(5,5),(1,5)\}$







4.11. Feladat megoldása.

(a) $\rho^+ = \{(a,b) \in A^2 \colon |a-b| \text{ is even}\}$, ahol $A = \{1,2,3,4,5\}$
(b) $\tau^+ = \tau$
(c) $\alpha^+ = \{(a,b) \in \mathbb{R}^2 \colon \exists k \in \mathbb{Z}^+ \colon b = a^{2k}\}$
(d) $\beta^+ = \{(x,y) \in \mathbb{R}^2 \colon \exists k \in \mathbb{Z}^+ \colon y - x = k\}$