1. feladatsor – Halmazok

Megoldások

1. Feladat.

- $A \cup B = \{a, b, c, d, e\} = U$;
- $A \cap B = \{d\}$;
- $\overline{B} = \{a, b, c\}$;

- $A \setminus B = \{a, b, c\}$;
- $A \triangle B = \{a, b, c, e\}$;

- $\left(A \triangle \overline{C}\right) \setminus \overline{B} = \emptyset$;
- $\mathcal{P}(B) = \{\emptyset, \{d\}, \{e\}, \{d, e\}\}$.

2. Feladat. $A = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$, $B = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

- $A \cup B = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$;
- $A \cap B = \{\emptyset, \{b\}\}$;
- $A \setminus B = \{\{a\}, \{a, b\}\}$;

- $B \setminus A = \{\{c\}, \{b, c\}\}$;
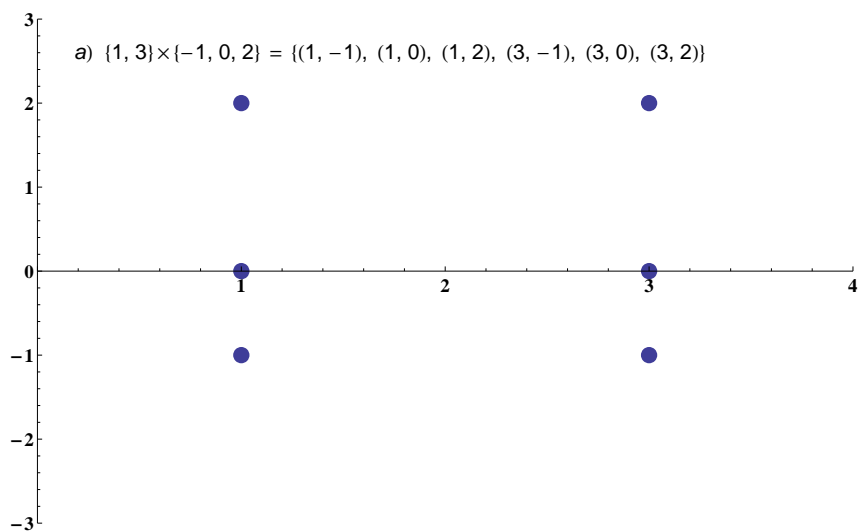- $A \triangle B = \{\{a\}, \{c\}, \{a, b\}, \{b, c\}\}$.

3. Feladat.

a) $\emptyset \in A$

b) $\emptyset \subseteq A$

c) $\{\emptyset\} \in A$

d) $\{\emptyset\} \subseteq A$

e) $\{\{\emptyset\}\} \notin A$

f) $\{\{\emptyset\}\} \subseteq A$

g) $\{\{\emptyset\}, \emptyset\} \in A$
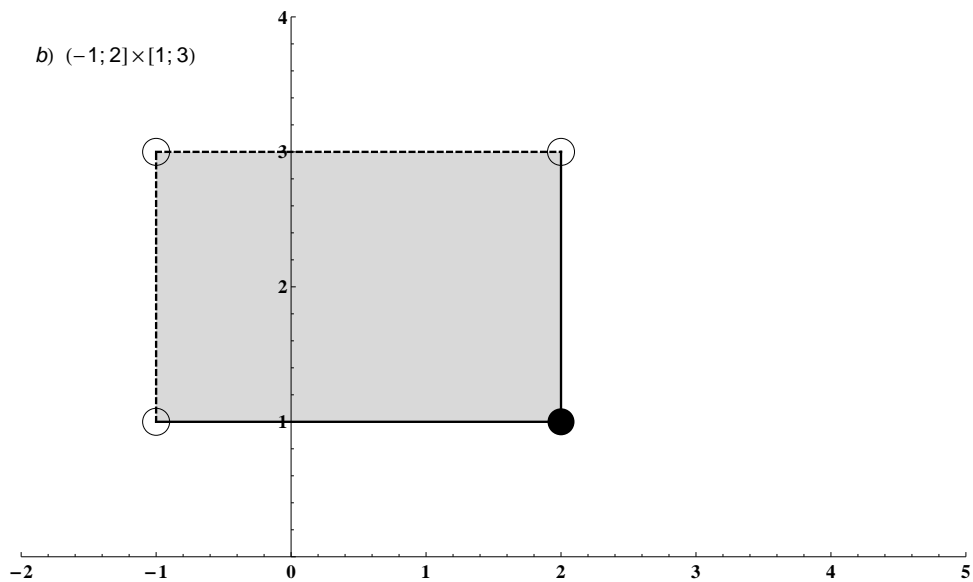
h) $\{\{\emptyset\}, \emptyset\} \subseteq A$

4. Feladat. $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$.

5. Feladat.

a) $A \setminus (B \setminus C) \neq (A \setminus B) \setminus C$

b) $(A \setminus B) \setminus B = A \setminus B$

c) $A \cap (B \cup C) \neq (A \cup B) \cap (A \cup C)$

d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

e) $A = (A \cup B) \setminus (B \setminus A)$

f) $(A \cap B) \setminus (B \setminus (A \cup C)) = A \cap B$

g) $(A \triangle B) \triangle (A \cap B) = A \cup B$

6. Feladat.

a) $\{1, 3\} \times \{-1, 0, 2\} = \{(1, -1), (1, 0), (1, 2), (3, -1), (3, 0), (3, 2)\}$

b) $(-1; 2] \times [1; 3]$

7. Feladat. $\overline{A \cup (B \cap (C \cup D))} = \overline{A} \cap \left(\overline{B} \cup \left(\overline{C} \cap \overline{D} \right) \right)$

8. Feladat. $A \subseteq B$ teljesül, $A = B$ és $B \setminus A = \emptyset$ nem teljesül.

9. Feladat.

a)

$$
\begin{aligned}
x \in (A \cap C) \cup (B \cap D) \quad &\longleftrightarrow \quad x \in (A \cup (B \cap D)) \cap (C \cup (B \cap D)) \\
&\longleftrightarrow \quad x \in (A \cup B) \cap (A \cup D) \cap (C \cup B) \cap (C \cup D) \\
&\longrightarrow \quad x \in (A \cup B) \cap (C \cup D)
\end{aligned}
$$

b)

$$x \in C \cup D \longrightarrow x \notin B \cap \overline{(C \cup D)} = B \setminus (C \cup D)$$

$$\nearrow$$

$$x \in A \cap C \cap D$$

$$\searrow$$

$$x \in A \cap C$$

$$\longrightarrow \quad x \in \quad (A \cap C) \setminus (B \setminus (C \cup D))$$

10. Feladat.

- $A \sqcap A = \overline{A \cap A} = \overline{A}$
- $(A \sqcap B) \sqcap (A \sqcap B) = \overline{\overline{A \sqcap B}} = \overline{\overline{A \cap B}} = A \cap B$
- $A \cup B = (A \sqcap A) \sqcap (B \sqcap B)$