Recovering a projective space from its point subset

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An affine space can be obtained from projective space by removing one of its hyperplanes. There are more examples where removing a hyperplane yields some affine geometries, e.g. in a polar space deleting its geometric hyperplane produces an affine polar space (cf. [1], [4]). When we delete any subspace in a projective space, then so called slit space arises, possessing both projective and affine properties (cf. [2], [3]). So, a general question appears: how big the remaining fragment of a projective space must be so as the surrounding space can be recovered in terms of the internal geometry of this fragment?

We deal with a projective space $\mathfrak{M}$, where any point subset $\mathcal{W}$, a horizon, is removed. Lines of $\mathfrak{M}$, that are entirely contained in $\mathcal{W}$, are also removed. In the remaining fragment of $\mathfrak{M}$, that we call a complement, we introduce a parallelism: two lines are parallel if they meet in the horizon $\mathcal{W}$. Two disjoint classes of maximal cliques of parallelism can be distinguished. Both of them are definable in terms of the complement. One of them, that we call a star direction, consists of lines passing through the fixed point in $\mathcal{W}$. We recover removed points by identifying them with star directions. Next we investigate planes of our complement. It turns out, that there is a triangle with sides, which are not totally removed, on every such plane. Thus, all considered planes are definable in the complement. We use planes sections to recover missing lines. All our research are done under the following assumption:

($*$) on every line in $\mathfrak{M}$ the number of removed points is less or equal to the number of remaining points minus two.

Finally we prove

Theorem. *If $\mathfrak{M}$ is a projective space of a dimension at least 3, $\mathcal{W}$ is a point subset of $\mathfrak{M}$, and $\mathfrak{M}$ satisfies ($*$), then both $\mathfrak{M}$ and $\mathcal{W}$ can be recovered in the complement of $\mathcal{W}$ in $\mathfrak{M}$.*

References

[1] Cohen, A. M., and Shult, E. E. *Affine polar spaces*, Geom. Dedicata **35** (1990), 43–76.

[2] Karzel, H., and Meissner, H. *Geschlitze Inzidenzgruppen und normale Fastmoduln*, Abh. Math. Sem. Univ. Hamb., **31** (1967), 69–88.

[3] Karzel, H., and Pieper, I. *Bericht über geschlitzte Inzidenzgruppen*, Jber. Deutsh. Math.-Verein. **70** (1970), 70–114.

[4] Prażmowska, M., Prażmowski, K., and Żynel, M. *Affine polar spaces, their Grassmannians, and adjacencies*, Math. Pannonica **20** (2009), 37–59.