1. Let $(X,<)$ be an arbitrary ordered set. Show that the elements of X can be colored by two colors in such a way that between any two points of the same color there is a point of the opposite color.

2. Let p be a prime and let M be an $n \times m$ matrix with integer entries such that $Mv \not\equiv 0 \pmod{p}$ for any column vector $v \neq 0$ whose entries are 0 or 1. Show that there exists a row vector x with integer entries such that no entry of xM is $0 \pmod{p}$.

3. Let $Z = \{z_1, \ldots, z_{n-1}\}$, $n \geq 2$, be a set of different complex numbers such that Z contains the conjugate of any its element.

a) Show that there exists a constant C, depending on Z, such that for any $\varepsilon \in (0,1)$ there exists an algebraic integer x_0 of degree n, whose algebraic conjugates $x_1, x_2, \ldots, x_{n-1}$ satisfy $|x_1 - z_1| \leq \varepsilon, \ldots, |x_{n-1} - z_{n-1}| \leq \varepsilon$ and $|x_0| \leq C/\varepsilon$.

b) Show that there exists a set $Z = \{z_1, \ldots, z_{n-1}\}$ and a positive number c_n such that for any algebraic integer x_0 of degree n, whose algebraic conjugates satisfy $|x_1 - z_1| \leq \varepsilon, \ldots, |x_{n-1} - z_{n-1}| \leq \varepsilon$, it also holds that $|x_0| > c_n/\varepsilon$.

4. Let $\{a_{n,1}, \ldots, a_{n,n}\}_{n=1}^{\infty}$ integers such that $a_{n,i} \neq a_{n,j}$ for $1 \leq i < j \leq n$, $n = 2, 3, \ldots$ and let $\langle y \rangle \in [0,1)$ denote the fractional part of the real number y. Show that there exists a real sequence $\{x_n\}_{n=1}^{\infty}$ such that the numbers $\langle a_{n,1}x_n \rangle, \ldots, \langle a_{n,n}x_n \rangle$ are asymptotically uniformly distributed on the interval $[0,1]$.

5. Let $d > 1$ be integer and $0 < r < 1/2$. Show that there exist finitely many (depending only on d, r) nonzero vectors in $\mathbf{R}^d$ such that if the distance of a straight line in $\mathbf{R}^d$ from the integer lattice $\mathbf{Z}^d$ is at least r, then this line is orthogonal to one of these finitely many vectors.

6. Show that the recursion $n = x_n(x_{n-1} + x_n + x_{n+1})$, $n = 1, 2, \ldots$, $x_0 = 0$ has exactly one nonnegative solution.

7. Let r be a nonnegative continuous function on the real line. Show that there exists a function $f \in C^1(\mathbf{R})$, not identically zero, such that $f'(x) = f(x - r(f(x)))$, $x \in \mathbf{R}$.

8. Let $f_1, f_2, \dots$ be continuous real functions on the real line. Is it true that if the series $\sum_{n=1}^{\infty} f_n(x)$ is divergent for every x, then this holds also true for any typical choice of the signs in the sum (i.e. the set of those $\{\epsilon_n\}_{n=1}^{\infty} \in \{+1, -1\}^{\mathbf{N}}$ sequences, for which the series $\sum_{n=1}^{\infty} \epsilon_n f_n(x)$ is convergent at least at one point x, forms a subset of first category within the set $\{+1, -1\}^{\mathbf{N}}$)?

9. Given finitely many open half planes on the Euclidean plane. The boundary lines of these half planes divide the plane into convex domains. Find a polynomial $C(q)$ of degree two so that the following holds: for any $q \geq 1$ integer, if the half planes cover each point of the plane at least q times, then the set of points covered exactly q times is the union of at most $C(q)$ domains.

10. Let X and Y be independent random variables with "Saint-Petersburg" distribution, i.e. for any $k = 1, 2, \dots$ their value is 2^k with probability $1/2^k$. Show that X and Y can be realized on a sufficiently big probability space such that there exists another pair of independent "Saint-Petersburg" random variables (X', Y') on this space with the property that $X + Y = 2X' + Y' I(Y' \leq X')$ almost surely (here $I(A)$ denotes the indicator function of the event A).

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Unofficial translation by L. Erdős