1. Feladatsor - megoldások

1.1. Feladat. Legyen

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & -2 & 3 \end{pmatrix},\ B = \begin{pmatrix} 1 & 2 \\ -1 & 1 \\ 1 & 3 \end{pmatrix},\ C = \begin{pmatrix} 1 & 2 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix},\ E = \begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix},\ F = \begin{pmatrix} 1 & -2 & 1 \\ 1 & 1 & 1 \\ -1 & 2 & 1 \end{pmatrix}.$$

Számítsuk ki az alábbi mátrixszorzatok közül a létezőket.

$$AB = \begin{pmatrix} -2 & 1 \\ 6 & 9 \end{pmatrix}, BA = \begin{pmatrix} 3 & -2 & 5 \\ 0 & -4 & 4 \\ 4 & -4 & 8 \end{pmatrix}, CB = \begin{pmatrix} -1 & 4 \end{pmatrix}$$

$$BC \text{ nem létezik }, DC = \begin{pmatrix} 1 & 2 & 0 \\ -1 & -2 & 0 \\ 2 & 4 & 0 \end{pmatrix}, CD = \begin{pmatrix} -1 \end{pmatrix}$$

$$EB^T = \begin{pmatrix} 5 & 1 & 7 \\ 3 & 3 & 5 \end{pmatrix}, BF \text{ nem létezik }, E^T A = \begin{pmatrix} 0 & 4 & -4 \\ 4 & 0 & 4 \end{pmatrix}$$

$$F^2 = \begin{pmatrix} -2 & -2 & 0 \\ 1 & 1 & 3 \\ 0 & 6 & 2 \end{pmatrix}, D^T C^T = \begin{pmatrix} -1 \end{pmatrix}$$

$$(A+B)C \text{ nem létezik }, (A+B^T)C = \begin{pmatrix} 1 \\ 16 \end{pmatrix}$$

$$AD + B^T D = \begin{pmatrix} 1 \\ 16 \end{pmatrix}$$

1.2. Feladat.

$$A^2 + 3A - 4E =$$
$$= \begin{pmatrix} -2 & -8 & 3 \\ 14 & 22 & -20 \\ 13 & 9 & -5 \end{pmatrix} + \begin{pmatrix} 3 & 9 & -15 \\ 12 & -6 & 18 \\ 9 & 3 & 6 \end{pmatrix} + \begin{pmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{pmatrix} =$$
$$= \begin{pmatrix} -3 & 1 & -12 \\ 26 & 12 & -2 \\ 22 & 12 & -3 \end{pmatrix}.$$

1.3. Feladat.

$$\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}, \text{ ahol } a, b \in \mathbb{R}$$

1.4. Feladat. Mátrixok blokkos szorzása:

(a) $$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 4 & -1 & 2 & 0 \\ \hline 0 & 1 & 0 & 4 \end{array}\right) \cdot \left(\begin{array}{c|cc} 1 & 1 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 4 \\ \hline 2 & 0 & 0 \end{array}\right)$$

Jelöljük meg a mátrixok blokkjait:

$$\left(\begin{array}{rrr|r} 1 & 3 & 2 & 0 \\ 4 & -1 & 2 & 0 \\ \hline 0 & 1 & 0 & 4 \end{array}\right) = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad \left(\begin{array}{r|rr} 1 & 1 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 4 \\ \hline 2 & 0 & 0 \end{array}\right) = \begin{pmatrix} E & F \\ G & H \end{pmatrix}$$

Ekkor a szorzat:

$$\begin{pmatrix} AE+BG & AF+BH \\ DE+DG & CF+DH \end{pmatrix},$$

vagyis

$$\begin{pmatrix} \begin{pmatrix} 5 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 7 & 12 \\ 2 & 11 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 2 \end{pmatrix} + \begin{pmatrix} 8 \end{pmatrix} & \begin{pmatrix} 2 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \end{pmatrix} \end{pmatrix} =$$

$$\left(\begin{array}{r|rr} 5 & 7 & 12 \\ 0 & 2 & 11 \\ \hline 10 & 2 & 1 \end{array}\right).$$

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(b) $\left(\begin{array}{r|rr|rr} 1 & -1 & 2 & 0 & 1 \\ -1 & 2 & 0 & 1 & 0 \\ \hline 0 & 1 & 2 & 0 & 0 \end{array}\right) \cdot \left(\begin{array}{rr|rr} -1 & 2 & 2 & 1 \\ \hline 2 & -1 & 2 & 0 \\ 0 & 0 & 1 & 1 \\ \hline -2 & 1 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{array}\right)$

Jelöljük meg a mátrixok blokkjait:

$$\left(\begin{array}{r|rr|rr} 1 & -1 & 2 & 0 & 1 \\ -1 & 2 & 0 & 1 & 0 \\ \hline 0 & 1 & 2 & 0 & 0 \end{array}\right) = \begin{pmatrix} A & B & C \\ D & E & F \end{pmatrix}, \quad \begin{pmatrix} G & H \\ I & J \\ K & L \end{pmatrix}$$

Ekkor a szorzat:

$$\begin{pmatrix} AG+BI+CK & AH+BJ+CL \\ DG+EI+FK & DH+EJ+FL \end{pmatrix},$$

vagyis

$$\begin{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix} + \begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ -2 & 1 \end{pmatrix} & \begin{pmatrix} 2 & 1 \\ -2 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 4 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \end{pmatrix} + \begin{pmatrix} 2 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \end{pmatrix} \end{pmatrix} =$$

$$\left(\begin{array}{rr|rr} -1 & 3 & 2 & 4 \\ 3 & -3 & 3 & -1 \\ \hline 2 & -1 & 4 & 2 \end{array}\right).$$

1.5. Feladat. A 2. esetben nem...