

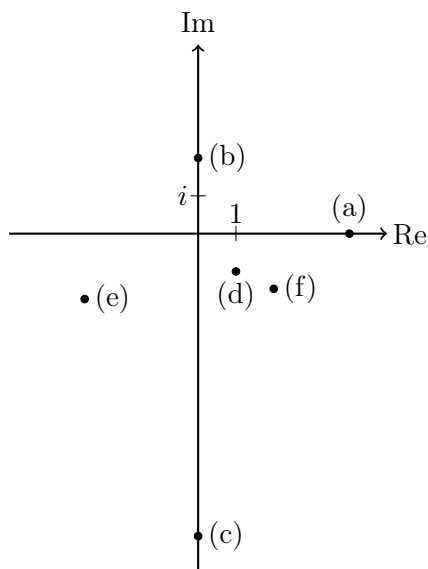
1. feladatsor – Komplex számok MEGOLDÁSOK

1.1. Feladat. A műveletek eredménye:

- (a) $1; -1;$
- (b) $2 - 4i; \quad 3 + 2i$
- (c) $32 - 7i;$
- (d) $-\frac{8}{13} + \frac{1}{13}i;$
- (e) $\sqrt{10};$
- (f) $\frac{17}{10} + \frac{19}{10}i.$

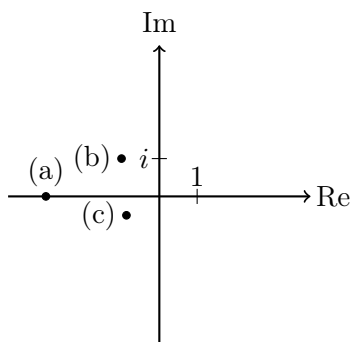
1.2. Feladat. Trigonometrikus alakok:

- (a) $4(\cos \pi + i \sin \pi);$
- (b) $2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2});$
- (c) $8(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2});$
- (d) $\sqrt{2}(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4});$
- (e) $\sqrt{12}(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6});$
- (f) $4(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}).$



1.3. Feladat. Kanonikus alakok:

- (a) $-3;$
- (b) $-1 + i;$
- (c) $-\frac{\sqrt{3}}{2} - \frac{1}{2}i.$



1.4. Feladat. A műveletek eredménye:

- (a) $8(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$;
- (b) $\frac{\sqrt{2}}{8}(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$;
- (c) $(\sqrt{2})^{11}(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4})$;
- (d) $2^{67}(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$;
- (e) $\frac{1}{256}(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3})$;
- (f) $2^{611}(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2})$.

1.5. Feladat. A gyökök trigonometrikus alakja:

- (a) $z_k = 2(\cos \alpha_k + i \sin \alpha_k)$, $\alpha_k = \frac{\pi}{2} + \frac{k \cdot 2\pi}{3}$, $k = 0, 1, 2$
 $z_0 = 2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}) = 2i$
 $z_1 = 2(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}) = -\sqrt{3} - i$
 $z_2 = 2(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}) = \sqrt{3} - i$
- (b) $z_k = 2(\cos \beta_k + i \sin \beta_k)$, $\beta_k = \frac{\pi}{4} + \frac{k \cdot \pi}{2}$, $k = 0, 1, 2, 3$
 $z_0 = 2(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}) = \sqrt{2} + \sqrt{2}i$
 $z_1 = 2(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}) = -\sqrt{2} + \sqrt{2}i$
 $z_2 = 2(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}) = -\sqrt{2} - \sqrt{2}i$
 $z_3 = 2(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}) = \sqrt{2} - \sqrt{2}i$;
- (c) $z_k = 2(\cos \gamma_k + i \sin \gamma_k)$, $\gamma_k = \frac{\pi}{3} + \frac{k \cdot 2\pi}{3}$, $k = 0, 1, 2$
 $z_0 = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}) = 1 + \sqrt{3}i$
 $z_1 = 2(\cos \pi + i \sin \pi) = -2$
 $z_2 = 2(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}) = 1 - \sqrt{3}i$;
- (d) $z_k = \sqrt[4]{2}(\cos \varepsilon_k + i \sin \varepsilon_k)$, $\varepsilon_k = \frac{\pi}{3} + \frac{k \cdot \pi}{2}$, $k = 0, 1, 2, 3$
 $z_0 = \sqrt[4]{2}(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}) = \frac{\sqrt[4]{2}}{2} + \frac{\sqrt[4]{18}}{2}i$
 $z_1 = \sqrt[4]{2}(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}) = -\frac{\sqrt[4]{18}}{2} + \frac{\sqrt[4]{2}}{2}i$
 $z_2 = \sqrt[4]{2}(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}) = -\frac{\sqrt[4]{2}}{2} - \frac{\sqrt[4]{18}}{2}i$
 $z_3 = \sqrt[4]{2}(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}) = \frac{\sqrt[4]{18}}{2} - \frac{\sqrt[4]{2}}{2}i$.

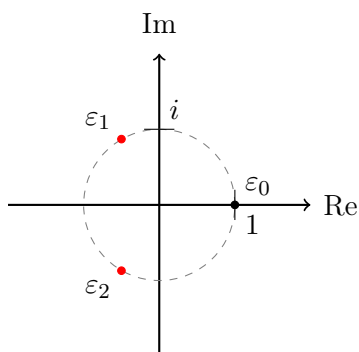
1.6. Feladat. Megoldások:

- (a) $\frac{z}{w} = -\frac{1+\sqrt{3}}{4} + \frac{\sqrt{3}-1}{4}i$
 $w^4 = -64$
 $\sqrt[6]{z} : \sqrt[6]{2}(\cos \alpha_k + i \sin \alpha_k)$, $\alpha_k = \frac{5\pi}{18} + \frac{k \cdot \pi}{3}$, $k = 0, 1, 2, 3, 4, 5$;

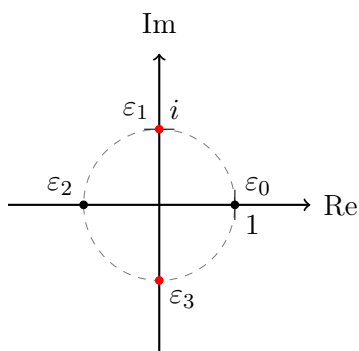
- (b) $\frac{z}{w} = -\frac{1+\sqrt{3}}{6} + \frac{\sqrt{3}-1}{6}i$
 $w^4 = -324$
 $\sqrt[6]{z} : \sqrt[6]{2}(\cos \beta_k + i \sin \beta_k), \quad \beta_k = \frac{\pi}{9} + \frac{k\pi}{3}, \quad k = 0, 1, 2, 3, 4, 5;$
- (c) $\frac{z}{w} = \frac{1-\sqrt{3}}{4} - \frac{1+\sqrt{3}}{4}i$
 $w^6 = 512i$
 $\sqrt[5]{z} : \sqrt[5]{2}(\cos \gamma_k + i \sin \gamma_k), \quad \gamma_k = \frac{\pi}{30} + \frac{k\cdot 2\pi}{5}, \quad k = 0, 1, 2, 3, 4;$
- (d) $\frac{z}{w} = -1 + \sqrt{3} - (1 + \sqrt{3})i$
 $w^5 = 4 - 4i$
 $\sqrt[6]{z} : \sqrt[3]{2}(\cos \delta_k + i \sin \delta_k), \quad \delta_k = \frac{\pi}{18} + \frac{k\pi}{3}, \quad k = 0, 1, 2, 3, 4, 5;$

1.7. Feladat. Primitív egységgyököket pirossal jelöltük. Az n -edik egységgyökök esetén: $\varepsilon_k = \cos(k \cdot \frac{2\pi}{n}) + i \sin(k \cdot \frac{2\pi}{n})$, ahol $k = 0, 1, \dots, n-1$.

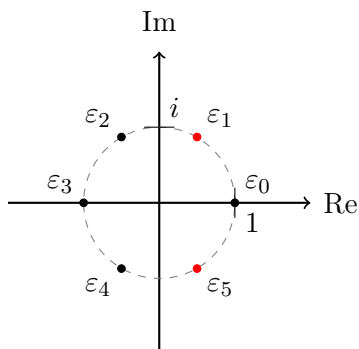
(a) harmadik egységgyökök:



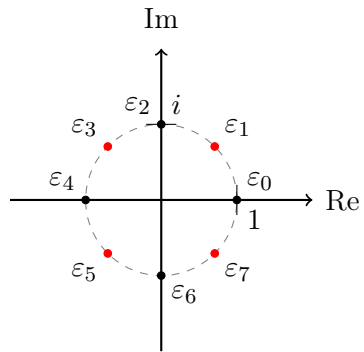
(b) negyedik egységgyökök:



(c) hatodik egységgyökök:



(d) nyolcadik egységgyökök:



1.8. Feladat. Megoldások:

- (a) nem egységgyök;
- (b) egységgyök, de nem primitív;
- (c) egységgyök, de nem primitív;
- (d) nem egységgyök;
- (e) primitív egységgyök:

$$z^0 = 1$$

$$z^2 = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}$$

$$z^3 = \cos \frac{10\pi}{7} + i \sin \frac{10\pi}{7}$$

$$z^4 = \cos \frac{4\pi}{7} + i \sin \frac{4\pi}{7}$$

$$z^5 = \cos \frac{12\pi}{7} + i \sin \frac{12\pi}{7}$$

$$z^6 = \cos \frac{6\pi}{7} + i \sin \frac{6\pi}{7}$$