
DISZKRÉT MATEMATIKA I.

6. FELADATSOR

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A 6. feladatsor feladatainak megoldása

6.1. Feladat.

- (a) $-i$, (b) -1 , (c) $41 - 11i$, (d) $17 - 2i$,
(e) $-15 - 5i$, (f) $-\frac{11}{17} + \frac{27}{17}i$, (g) $-\frac{3}{13} - \frac{11}{13}i$, (h) $\frac{11}{10} - \frac{23}{10}i$,
(i) $-\frac{2}{5} + \frac{3}{10}i$.

↪ videó: [6.1. Feladat: \(a\)–\(e\)](#), [6.1. Feladat: \(f\)–\(i\)](#)

6.2. Feladat.

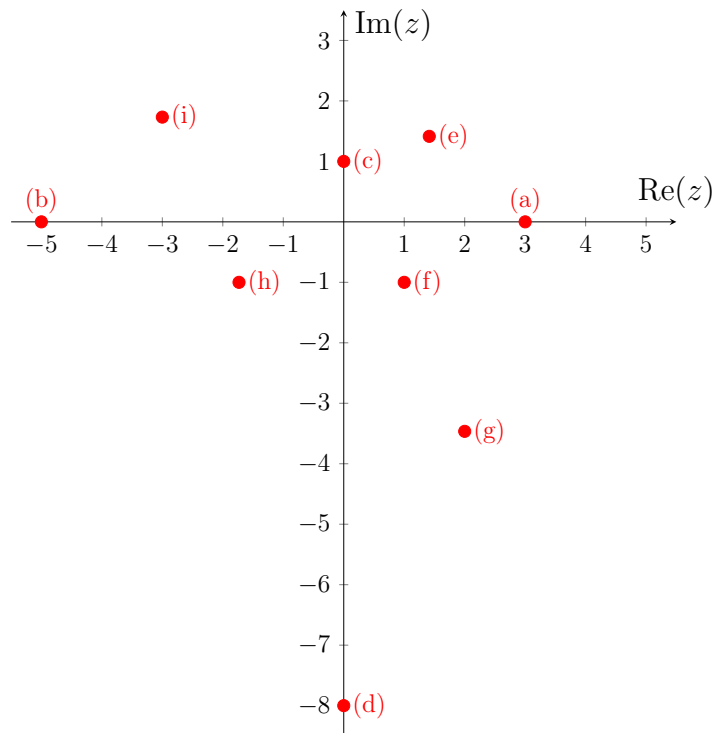
- (a) $z = 4 + 3i$;
(b) $z = -\frac{3}{13} - \frac{11}{13}i$;
(c) $z_1 = -2 - \frac{3}{2}i$, $z_2 = 2 + \frac{3}{2}i$;
(d) $z_1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$, $z_2 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$;
(e) $z_1 = -1 - 4i$, $z_2 = 1 + 4i$;
(f) $z_1 = 3 - 2i$, $z_2 = -3 + 2i$.

↪ videó: [6.2. Feladat: \(a\), \(c\) és \(e\)](#)

6.3. Feladat.

- (a) $3 = 3 \cdot (\cos 0 + i \cdot \sin 0) = 3e^0$, (b) $-5 = 5 \cdot (\cos \pi + i \cdot \sin \pi) = 5e^{\pi i}$,
(c) $i = \cos \frac{\pi}{2} + i \cdot \sin \frac{\pi}{2} = e^{\frac{\pi}{2}i}$, (d) $-8i = 8(\cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2}) = 8e^{\frac{3\pi}{2}i}$,
(e) $\sqrt{2} + \sqrt{2}i = 2(\cos \frac{\pi}{4} + i \cdot \sin \frac{\pi}{4}) = 2e^{\frac{\pi}{4}i}$, (f) $1 - i = \sqrt{2}(\cos \frac{7\pi}{4} + i \cdot \sin \frac{7\pi}{4}) = \sqrt{2}e^{\frac{7\pi}{4}i}$,
(g) $2 - 2\sqrt{3}i = 4(\cos \frac{5\pi}{3} + i \cdot \sin \frac{5\pi}{3}) = 4e^{\frac{5\pi}{3}i}$, (h) $-\sqrt{3} - i = 2(\cos \frac{7\pi}{6} + i \cdot \sin \frac{7\pi}{6}) = 2e^{\frac{7\pi}{6}i}$,
(i) $-3 + \sqrt{3}i = 2\sqrt{3}(\cos \frac{5\pi}{6} + i \cdot \sin \frac{5\pi}{6}) = 2\sqrt{3}e^{\frac{5\pi}{6}i}$.

↪ videó: [6.3. Feladat \(a\), \(c\) és \(f\)](#), [6.3. Feladat: \(h\)](#)



6.4. Feladat.

(a) $2(\cos 0 + i \sin 0) = 2,$

(c) $2e^{\frac{\pi}{4}i} = \sqrt{2} + \sqrt{2}i,$

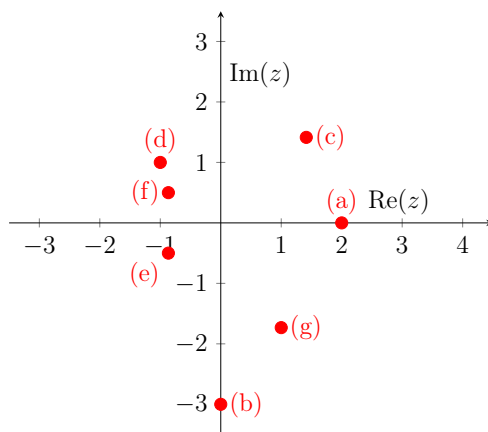
(e) $\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i,$

(g) $2(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}) = 1 - \sqrt{3}i.$

(b) $3e^{\frac{3\pi}{2}i} = -3i,$

(d) $\sqrt{2}(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}) = -1 + i,$

(f) $e^{\frac{5\pi}{6}i} = -\frac{\sqrt{3}}{2} + \frac{1}{2}i,$



↪ videó: [6.4. Feladat: \(a\)–\(b\)](#), [6.4. Feladat: \(d\) és \(f\)](#)

6.5. Feladat.

(a) $8(\cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2}) = -8i,$

(b) $20(\cos 0 + i \cdot \sin 0) = 20,$

- (c) $\cos \pi + i \cdot \sin \pi = -1$,
 (d) $2^{67}(\cos \frac{5\pi}{6} + i \cdot \sin \frac{5\pi}{6}) = -2^{66}\sqrt{3} + 2^{66}i$,
 (e) $2^{611}(\cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2}) = -2^{611}i$,
 (f) $6^{1526}(\cos \frac{2\pi}{3} + i \cdot \sin \frac{2\pi}{3}) = -3 \cdot 6^{1525} + 3\sqrt{3} \cdot 6^{1525}i$.

↪ videó: [6.5. Feladat: \(a\)](#), [6.5. Feladat: \(d\)](#)

6.6. Feladat.

- (a) $3 = 3(\cos 0 + i \sin 0)$, $-3 = 3(\cos \pi + i \sin \pi)$,
 (b) $2i = 2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$, $-2i = 2(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2})$,
 (c) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = \cos \frac{\pi}{4} + i \sin \frac{\pi}{4}$, $-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i = \cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}$,
 (d) $-2\sqrt{2} + 2\sqrt{2}i = 4(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4})$, $2\sqrt{2} - 2\sqrt{2}i = 4(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4})$,
 (e) $-1 - 4i$, $1 + 4i$,
 (f) $\sqrt{3} + i = 2(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$, $-\sqrt{3} - i = 2(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6})$,
 (g) $-2 = 2(\cos \pi + i \sin \pi)$, $1 + \sqrt{3}i = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$, $1 - \sqrt{3}i = 2(\cos(-\frac{\pi}{3}) + i \sin(-\frac{\pi}{3}))$,
 (h) $2i = 2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$, $-\sqrt{3} - i = 2(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6})$, $\sqrt{3} - i = 2(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6})$,
 (i) trigonometrikus alakban: $2\sqrt[6]{2} \left(\cos \frac{3\pi+2k\pi}{3} + i \sin \frac{3\pi+2k\pi}{3} \right)$ ($k = 0, 1, 2$), kanonikus alakban
 rendre: $\sqrt{2}\sqrt[6]{2} + \sqrt{2}\sqrt[6]{2}i$, $-\frac{\sqrt[6]{2}(1+\sqrt{3})}{\sqrt{2}} + \frac{\sqrt[6]{2}(-1+\sqrt{3})}{\sqrt{2}}i$, $\frac{\sqrt[6]{2}(-1+\sqrt{3})}{\sqrt{2}} - \frac{\sqrt[6]{2}(1+\sqrt{3})}{\sqrt{2}}i$.

↪ videó: [6.6. Feladat: \(b\)](#), [6.6. Feladat: \(h\)](#)

6.7. Feladat.

- (a) $x_1 = -i$, $x_2 = i$
 $x^2 + 1 = (x + i)(x - i)$
 (b) $x_1 = -3 - i$, $x_2 = -3 + i$
 $x^2 + 6x + 10 = (x + 3 + i)(x + 3 - i)$
 (c) $x_{1,2} = -i$
 $x^2 + 2xi - 1 = (x + i)^2$
 (d) $x_1 = -2$, $x_2 = 1 + \sqrt{3}i$, $x_3 = 1 - \sqrt{3}i$
 $x^3 + 8 = (x + 2)(x - 1 - \sqrt{3}i)(x - 1 + \sqrt{3}i)$
 (e) $x_{1,2} = 0$, $x_3 = -3i$, $x_4 = 3i$
 $x^4 + 9x^2 = x^2(x + 3i)(x - 3i)$
 (f) $x_1 = -2$, $x_2 = 2$, $x_3 = -2i$, $x_4 = 2i$
 $x^4 - 16 = (x + 2)(x - 2)(x + 2i)(x - 2i)$
 (g) $x_{1,2} = -3i$, $x_{3,4} = 3i$
 $x^4 + 18x^2 + 81 = (x + 3i)^2(x - 3i)^2$

↪ videó: [6.7. Feladat: \(b\) és \(g\)](#)

6.8. Feladat. (a) x^2 , (b) $-x^2 + 8x - 4$, (c) $-\frac{13}{15}x^3 + 3x^2 + \frac{28}{15}x$.

↪ videó: [6.8. Feladat: \(b\)](#)

Szabaduló szoba