6. feladatsor – Komplex számok, polinomok

6.1. Feladat megoldása.

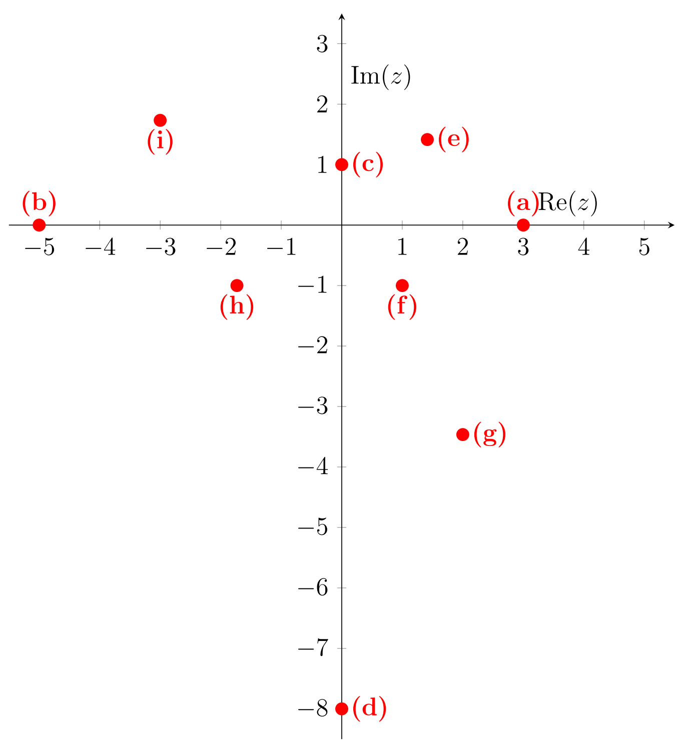
(a) $-i$;
(b) -1;
(c) $41 - 11i$;
(d) $17 - 2i$;
(e) $-15 - 5i$;
(f) $-\frac{11}{17} + \frac{27}{17}i$;
(g) $-\frac{3}{13} - \frac{11}{13}i$;
(h) $\frac{11}{10} - \frac{23}{10}i$;
(i) $-\frac{2}{5} + \frac{3}{10}i$.

6.2. Feladat megoldása.

(a) $z = 4 + 3i$;
(b) $z = -\frac{3}{13} - \frac{11}{13}i$;
(c) $z_1 = -2 - \frac{3}{2}i$, $z_2 = 2 + \frac{3}{2}i$;
(d) $z_1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$, $z_2 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$;
(e) $z_1 = -1 - 4i$, $z_2 = 1 + 4i$;
(f) $z_1 = 3 - 2i$, $z_2 = -3 + 2i$.

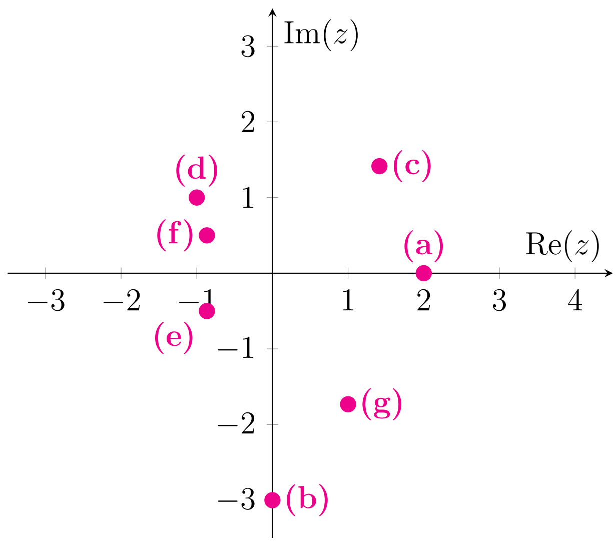
6.3. Feladat megoldása.

(a) $3 = 3 \cdot (\cos 0 + i \cdot \sin 0) = 3e^0$;
(b) $-5 = 5 \cdot (\cos \pi + i \cdot \sin \pi) = 5e^{\pi i}$;
(c) $i = \cos \frac{\pi}{2} + i \cdot \sin \frac{\pi}{2} = e^{\frac{\pi}{2}i}$;
(d) $-8i = 8(\cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2}) = 8e^{\frac{3\pi}{2}i}$;
(e) $\sqrt{2} + \sqrt{2}i = 2(\cos \frac{\pi}{4} + i \cdot \sin \frac{\pi}{4}) = 2e^{\frac{\pi}{4}i}$;
(f) $1 - i = \sqrt{2}(\cos \frac{7\pi}{4} + i \cdot \sin \frac{7\pi}{4}) = \sqrt{2}e^{\frac{7\pi}{4}i}$;
(g) $2 - 2\sqrt{3}i = 4(\cos \frac{5\pi}{3} + i \cdot \sin \frac{5\pi}{3}) = 4e^{\frac{5\pi}{3}i}$;
(h) $-\sqrt{3} - i = 2(\cos \frac{7\pi}{6} + i \cdot \sin \frac{7\pi}{6}) = 2e^{\frac{7\pi}{6}i}$;
(i) $-3 + \sqrt{3}i = 2\sqrt{3}(\cos \frac{5\pi}{6} + i \cdot \sin \frac{5\pi}{6}) = 2\sqrt{3}e^{\frac{5\pi}{6}i}$.



6.4. Feladat megoldása.

(a) $2(\cos 0 + i \sin 0) = 2;$
(b) $3e^{\frac{3\pi}{2}i} = -3i;$
(c) $2e^{\frac{\pi}{4}i} = \sqrt{2} + \sqrt{2}i;$
(d) $\sqrt{2}(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}) = -1 + i;$
(e) $\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i;$
(f) $e^{\frac{5\pi}{6}i} = -\frac{\sqrt{3}}{2} + \frac{1}{2}i;$
(g) $2(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}) = 1 - \sqrt{3}i.$



6.5. Feladat megoldása.

(a) $8(\cos\frac{3\pi}{2} + i \cdot \sin\frac{3\pi}{2}) = -8i;$
(b) $20(\cos 0 + i \cdot \sin 0) = 20;$
(c) $\cos\pi + i \cdot \sin\pi = -1;$
(d) $2^{67}(\cos\frac{5\pi}{6} + i \cdot \sin\frac{5\pi}{6}) = -2^{66}\sqrt{3} + 2^{66}i;$
(e) $2^{611}(\cos\frac{3\pi}{2} + i \cdot \sin\frac{3\pi}{2}) = -2^{611}i;$
(f) $6^{1526}(\cos\frac{2\pi}{3} + i \cdot \sin\frac{2\pi}{3}) = -3 \cdot 6^{1525} + 3\sqrt{3} \cdot 6^{1525}i.$

6.6. Feladat megoldása.

(a) $3 = 3(\cos 0 + i\sin 0)$
$-3 = 3(\cos\pi + i\sin\pi)$
(b) $2i = 2(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$
$-2i = 2(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2})$
(c) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$
$-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}$
(d) $-2\sqrt{2} + 2\sqrt{2}i = 4(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4})$
$2\sqrt{2} - 2\sqrt{2}i = 4(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$
(e) $-1 - 4i,\ 1 + 4i$
(f) $\sqrt{3} + i = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$
$-\sqrt{3} - i = 2(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6})$
(g) $-2 = 2(\cos\pi + i\sin\pi)$
$1 + \sqrt{3}i = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$
$1 - \sqrt{3}i = 2(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3}))$
(h) $2i = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right),$
$-\sqrt{3} - i = 2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right),$
$\sqrt{3} - i = 2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right).$
(i) $\sqrt{2}\sqrt[6]{2} + \sqrt{2}\sqrt[6]{2}i = 2\sqrt[6]{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right),$
$2\sqrt[6]{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right),$
$2\sqrt[6]{2}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right).$

6.7. Feladat megoldása.

(a) $x_1 = -i$, $x_2 = i$
$x^2 + 1 = (x+i)(x-i)$

(b) $x_1 = -3 - i$, $x_2 = -3 + i$
$x^2 + 6x + 10 = (x+3+i)(x+3-i)$

(c) $x_{1,2} = -i$
$x^2 + 2xi - 1 = (x+i)^2$

(d) $x_1 = -2$, $x_2 = 1 + \sqrt{3}i$, $x_3 = 1 - \sqrt{3}i$
$x^3 + 8 = (x+2)(x-1-\sqrt{3}i)(x-1+\sqrt{3}i)$

(e) $x_{1,2} = 0$, $x_3 = -3i$, $x_4 = 3i$
$x^4 + 9x^2 = x^2(x+3i)(x-3i)$

(f) $x_1 = -2$, $x_2 = 2$, $x_2 = -2i$, $x_3 = 2i$
$x^4 - 16 = (x+2)(x-2)(x+2i)(x-2i)$

(g) $x_{1,2} = -3i$, $x_{3,4} = 3i$
$x^4 + 18x^2 + 81 = (x+3i)^2(x-3i)^2$

6.8. Feladat megoldása. **(a)** x^2; **(b)** $-x^2 + 8x - 4$; **(c)** $-\frac{13}{15}x^3 + 3x^2 + \frac{28}{15}x$.