6. feladatsor – Komplex számok, polinomok

6.1. Feladat megoldása.

(a) $-i$; -1; **(b)** $41-11i$; **(c)** $17-2i$; **(d)** $-\frac{11}{17}+\frac{27}{17}i$; **(e)** $-\frac{3}{13}-\frac{11}{13}i$; **(f)** $\frac{11}{10}-\frac{23}{10}i$; **(g)** $-\frac{2}{5}+\frac{3}{10}i$.

6.2. Feladat megoldása.

(a) $3=3\cdot(\cos 0+i\cdot\sin 0)=3e^0$;

(b) $-5=5\cdot(\cos\pi+i\cdot\sin\pi)=5e^{\pi i}$;

(c) $i=\cos\frac{\pi}{2}+i\cdot\sin\frac{\pi}{2}=e^{\frac{\pi}{2}i}$;

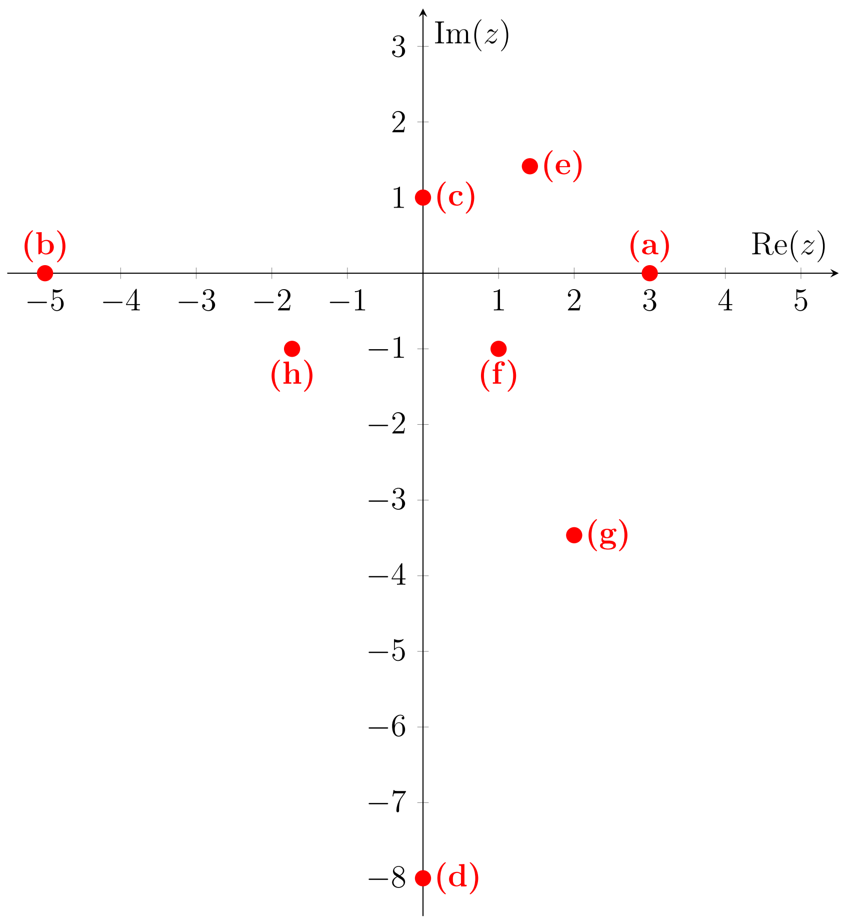
(d) $-8i=8(\cos\frac{3\pi}{2}+i\cdot\sin\frac{3\pi}{2})=8e^{\frac{3\pi}{2}i}$;

(e) $\sqrt{2}+\sqrt{2}i=2(\cos\frac{\pi}{4}+i\cdot\sin\frac{\pi}{4})=2e^{\frac{\pi}{4}i}$;

(f) $1-i=\sqrt{2}(\cos\frac{7\pi}{4}+i\cdot\sin\frac{7\pi}{4})=\sqrt{2}e^{\frac{7\pi}{4}i}$;

(g) $2-2\sqrt{3}i=4(\cos\frac{5\pi}{3}+i\cdot\sin\frac{5\pi}{3})=4e^{\frac{5\pi}{3}i}$;

(h) $-\sqrt{3}-i=2(\cos\frac{7\pi}{6}+i\cdot\sin\frac{7\pi}{6})=2e^{\frac{7\pi}{6}i}$.



6.3. Feladat megoldása.

(a) $2(\cos 0+i\sin 0)=2$;

(b) $3e^{\frac{3\pi}{2}i}=-3i$;

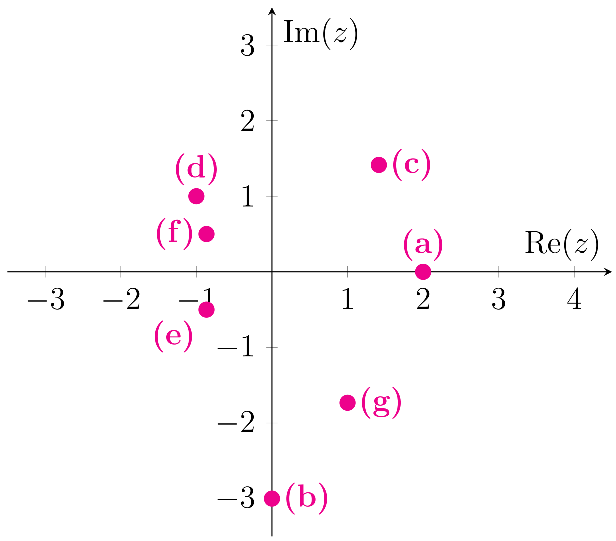
(c) $2e^{\frac{\pi}{4}i}=\sqrt{2}+\sqrt{2}i$;

(d) $\sqrt{2}(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})=-1+i$;

(e) $\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6}=-\frac{\sqrt{3}}{2}-\frac{1}{2}i$;

(f) $e^{\frac{5\pi}{6}i}=-\frac{\sqrt{3}}{2}+\frac{1}{2}i$;

(g) $2(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3})=1-\sqrt{3}i$.



6.4. Feladat megoldása.

(a) $8(\cos\frac{\pi}{6} + i \cdot \sin\frac{\pi}{6}) = 4\sqrt{3} + 4i$;
(b) $\cos\frac{3\pi}{2} + i \cdot \sin\frac{3\pi}{2} = -i$;
(c) $\cos(-\frac{\pi}{6}) + i \cdot \sin(-\frac{\pi}{6}) = \frac{\sqrt{3}}{2} - \frac{1}{2}i$;
(d) $\cos\pi + i \cdot \sin\pi = -1$;
(e) $2^{67}(\cos\frac{5\pi}{6} + i \cdot \sin\frac{5\pi}{6})$;
(f) $2^{611}(\cos\frac{3\pi}{2} + i \cdot \sin\frac{3\pi}{2})$;
(g) $6^{1526}(\cos\frac{2\pi}{3} + i \cdot \sin\frac{2\pi}{3})$.

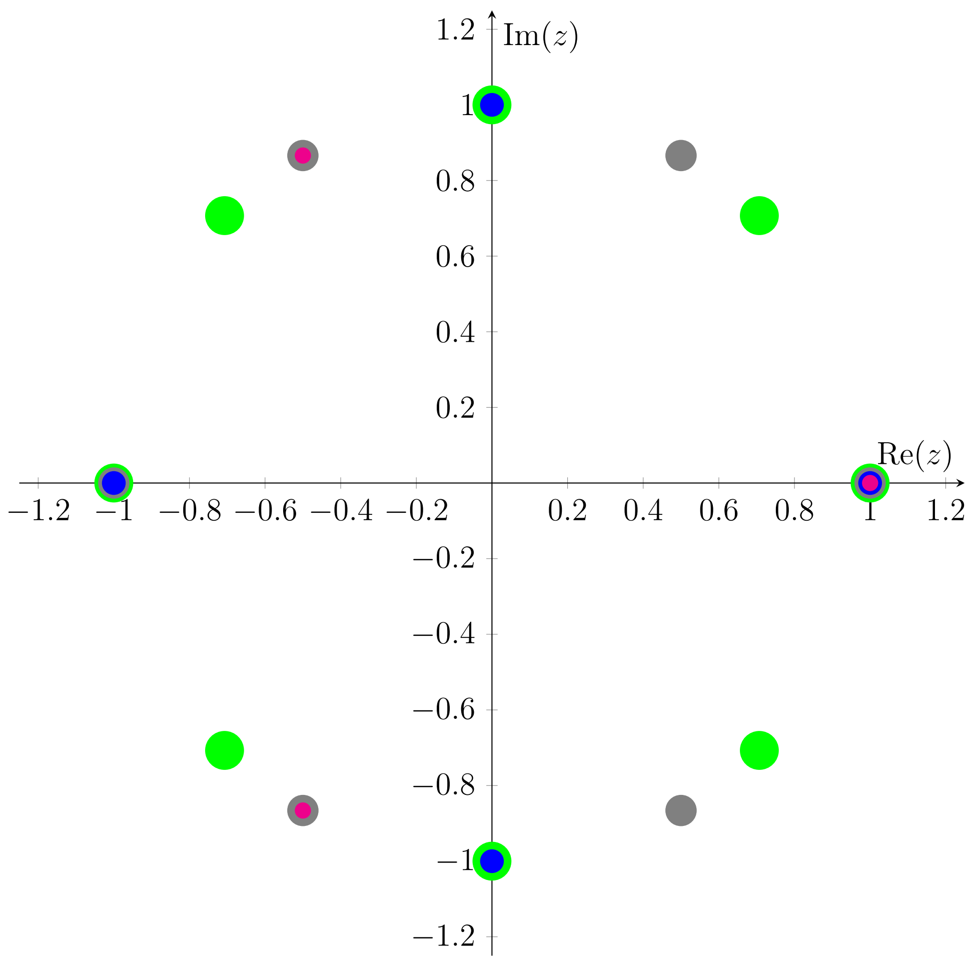
6.5. Feladat megoldása.

(a) $2i = 2(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$
$-2i = 2(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2})$
(b) $-2 = 2(\cos\pi + i\sin\pi)$
$1 + \sqrt{3}i = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$
$1 - \sqrt{3}i = 2(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3}))$
(c) $\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}$,
$\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}$,
$\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}$,
$\cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8}$.
(d) $2 = 2\left(\cos 0 + i\sin 0\right)$,
$1 + \sqrt{3}i = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$,
$-1 + \sqrt{3}i = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$,
$-2 = 2\left(\cos\pi + i\sin\pi\right)$,
$-1 - \sqrt{3}i = 2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$,
$1 - \sqrt{3}i = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$.
(e) $2i = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$,
$-\sqrt{3} - i = 2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$,
$\sqrt{3} - i = 2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$.
(f) $\sqrt[4]{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \sqrt[4]{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$,
$\sqrt[4]{2}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \sqrt[4]{2}\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$,
$\sqrt[4]{2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \sqrt[4]{2}\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$,
$\sqrt[4]{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \sqrt[4]{2}\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$.

6.6. Feladat megoldása.

(a) Harmadik egységgyökök: 1, $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$, $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$.
Primitív harmadik egységgyökök: $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$, $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$.
(b) Negyedik egységgyökök: 1, i, -1, $-i$.
Primitív negyedik egységgyökök: i, $-i$.
(c) Hatodik egységgyökök: 1, $\frac{1}{2} + \frac{\sqrt{3}}{2}i$, $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$, -1, $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$, $\frac{1}{2} - \frac{\sqrt{3}}{2}i$.
Primitív hatodik egységgyökök: $\frac{1}{2} - \frac{\sqrt{3}}{2}i$, $\frac{1}{2} + \frac{\sqrt{3}}{2}i$.

(d) Nyolcadik egységgyökök: $1,\ \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i,\ i,\ -\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i,\ -1,\ -\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i,\ -i,\ \frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i.$
Primitív nyolcadik egységgyökök: $\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i,\ -\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i,\ -\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i,\ \frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i.$



6.7. Feladat megoldása.

(a) $x_1=-3-i,\ x_2=-3+i$
$x^2+6x+10=(x+3+i)(x+3-i)$

(b) $x_1=2i,\ x_2=-\sqrt{3}-i,\ x_3=\sqrt{3}-i$
$2x^3+16i=(x-2i)(x+\sqrt{3}+i)(x-\sqrt{3}+i)$

(c) $x_1=-2,\ x_2=1+\sqrt{3}i,\ x_3=1-\sqrt{3}i$
$x^3+8=(x+2)(x-1-\sqrt{3}i)(x-1+\sqrt{3}i)$

(d) $x_1=\sqrt[4]{2}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=\sqrt[4]{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right),$

$x_2=\sqrt[4]{2}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)=\sqrt[4]{2}\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right),$

$x_3=\sqrt[4]{2}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=\sqrt[4]{2}\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right),$

$x_4=\sqrt[4]{2}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)=\sqrt[4]{2}\left(\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}\right).$

$x^4+1+\sqrt{3}i=(x-x_1)(x-x_2)(x-x_3)(x-x_4)$

6.8. Feladat megoldása.

(a) x^2 **(b)** $-x^2 + 8x - 4$ **(c)** $-\frac{13}{15}x^3 + 3x^2 + \frac{28}{15}x$