

## 4. feladatsor – Komplex számok, Polinomok

### Megoldások

#### 1. Feladat.

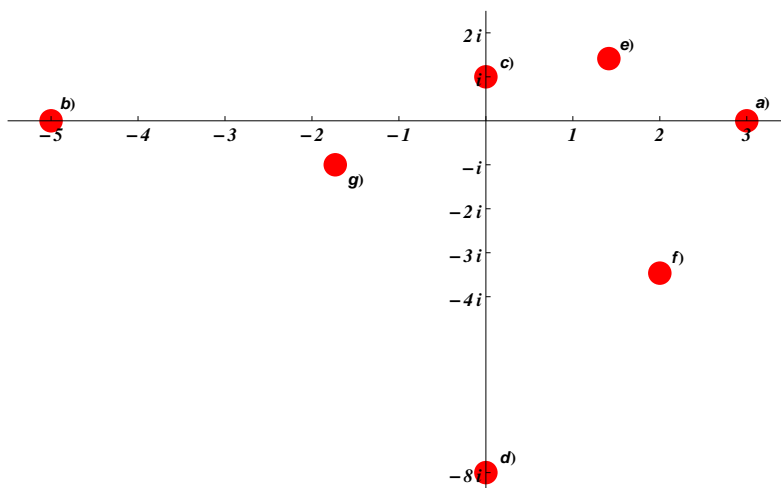
a)  $-i, -1$       b)  $41 - 11i$       c)  $17 - 2i$       d)  $-\frac{11}{17} + \frac{27}{17}i$       e)  $-\frac{3}{13} - \frac{11}{13}i$       f)  $\frac{11}{10} - \frac{23}{10}i$

#### 2. Feladat.

a)  $z = -\frac{13}{10} + \frac{11}{10}i$       d)  $z_1 = \frac{8}{3}, z_2 = 3 - i, z_3 = 3 + i$   
 b)  $z = 3 - i$       e)  $z_1 = 0, z_2 = -i, z_3 = \frac{\sqrt{3}}{2} + \frac{1}{2}i, z_4 = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$   
 c)  $z_1 = \frac{3}{2}, z_2 = 2 - i, z_3 = 2 + i$

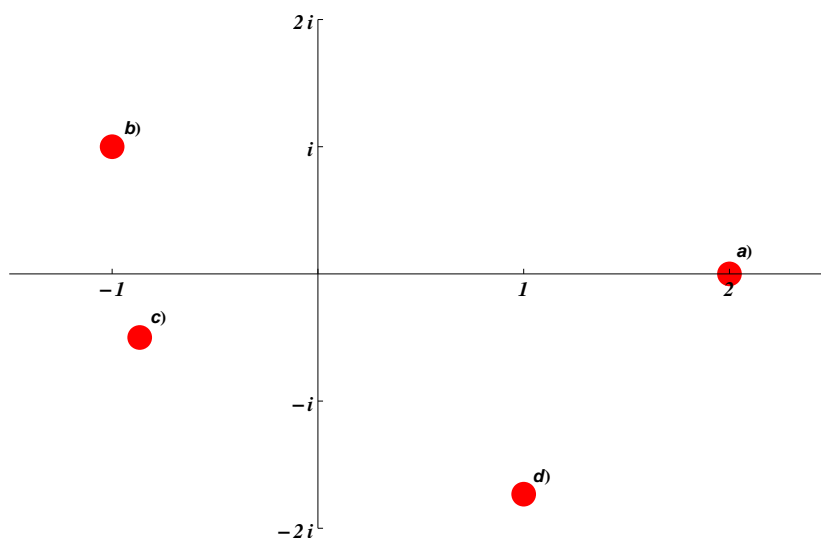
#### 3. Feladat.

a)  $3(\cos 0 + i \cdot \sin 0)$   
 b)  $5(\cos \pi + i \cdot \sin \pi)$   
 c)  $\cos \frac{\pi}{2} + i \cdot \sin \frac{\pi}{2}$   
 d)  $8(\cos(-\frac{\pi}{2}) + i \cdot \sin(-\frac{\pi}{2}))$   
 e)  $2(\cos \frac{\pi}{4} + i \cdot \sin(\frac{\pi}{4}))$   
 f)  $4(\cos(-\frac{\pi}{3}) + i \cdot \sin(-\frac{\pi}{3}))$   
 g)  $2(\cos(-\frac{5\pi}{6}) + i \cdot \sin(-\frac{5\pi}{6}))$



#### 4. Feladat.

a) 2  
 b)  $-1 + i$   
 c)  $-\frac{\sqrt{3}}{2} - \frac{1}{2}i$   
 d)  $1 - \sqrt{3}i$



### 5. Feladat.

- a)  $8 \left( \cos \frac{\pi}{6} + i \cdot \sin \frac{\pi}{6} \right) = 4\sqrt{3} + 4i$   
 b)  $\cos \left( -\frac{\pi}{2} \right) + i \cdot \sin \left( -\frac{\pi}{2} \right) = -i$   
 c)  $\cos \left( -\frac{\pi}{6} \right) + i \cdot \sin \left( -\frac{\pi}{6} \right) = \frac{\sqrt{3}}{2} - \frac{1}{2}i$

- d)  $2^{67} \left( \cos \frac{5\pi}{6} + i \cdot \sin \frac{5\pi}{6} \right) = -2^{66}\sqrt{3} + 2^{66}i$   
 e)  $2^{611} \left( \cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2} \right) = -2^{611}i$   
 f)  $6^{1526} \left( \cos \frac{2\pi}{3} + i \cdot \sin \frac{2\pi}{3} \right) = 6^{1525} (-3 + 3\sqrt{3}i)$

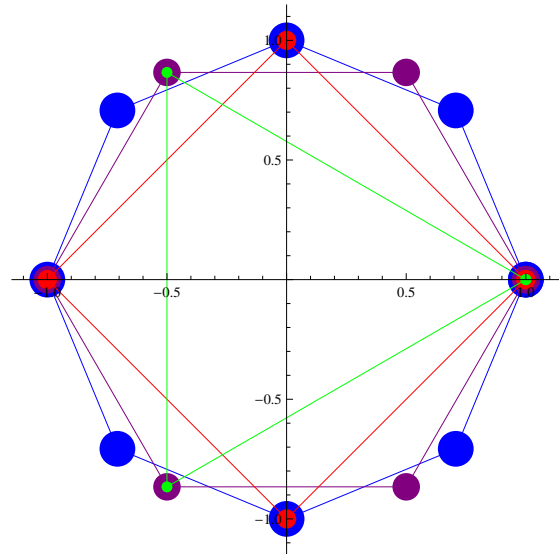
### 6. Feladat.

- a)  $\sqrt{2} + \sqrt{2}i = 2 \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right),$   
 $-\sqrt{2} + \sqrt{2}i = 2 \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right),$   
 $-\sqrt{2} - \sqrt{2}i = 2 \left( \cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right),$   
 $\sqrt{2} - \sqrt{2}i = 2 \left( \cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right).$   
 b)  $1 + \sqrt{3}i = 2 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right),$   
 $-2 = 2 \left( \cos \pi + i \sin \pi \right),$   
 $1 - \sqrt{3}i = 2 \left( \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right).$   
 c)  $\cos \frac{\pi}{8} + i \sin \frac{\pi}{8},$   
 $\cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8},$   
 $\cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8},$   
 $\cos \frac{13\pi}{8} + i \sin \frac{13\pi}{8}.$

- d)  $2 = 2 \left( \cos 0 + i \sin 0 \right),$   
 $1 + \sqrt{3}i = 2 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right),$   
 $-1 + \sqrt{3}i = 2 \left( \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right),$   
 $-2 = 2 \left( \cos \pi + i \sin \pi \right),$   
 $-1 - \sqrt{3}i = 2 \left( \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right),$   
 $1 - \sqrt{3}i = 2 \left( \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right).$   
 e)  $2i = 2 \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right),$   
 $-\sqrt{3} - i = 2 \left( \cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right),$   
 $\sqrt{3} - i = 2 \left( \cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right).$   
 f)  $\sqrt[4]{2} \left( \frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = \sqrt[4]{2} \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right),$   
 $\sqrt[4]{2} \left( -\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = \sqrt[4]{2} \left( \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right),$   
 $\sqrt[4]{2} \left( -\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) = \sqrt[4]{2} \left( \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right),$   
 $\sqrt[4]{2} \left( \frac{\sqrt{3}}{2} - \frac{1}{2}i \right) = \sqrt[4]{2} \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right).$

### 7. Feladat.

- a) Harmadik egységgyökök:  $1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i.$   
 Primitív harmadik egységgyökök:  $-\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i.$   
 b) Negyedik egységgyökök:  $1, i, -1, -i.$   
 Primitív negyedik egységgyökök:  $i, -i.$   
 c) Hatodik egységgyökök:  $1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -1,$   
 $-\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i.$   
 Primitív hatodik egységgyökök:  $\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} + \frac{\sqrt{3}}{2}i.$   
 d) Nyolcadik egységgyökök:  $1, \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, i, -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i,$   
 $-1, -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, -i, \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i.$   
 Primitív nyolcadik egységgyökök:  $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i,$   
 $-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i.$



### 8. Feladat.

- a)  $x_1 = -3 - i, x_2 = -3 + i, x^2 + 6x + 10 = (x + 3 + i)(x + 3 - i)$   
 b)  $x_1 = 1 + 2i, x_2 = 3 - 4i, x^2 - (4 - 2i)x + 11 + 2i = (x - 1 - 2i)(x - 3 + 4i)$   
 c)  $x_1 = 2i, x_2 = -\sqrt{3} - i, x_3 = \sqrt{3} - i, 2x^3 + 16i = 2(x - 2i)(x + \sqrt{3} + i)(x - \sqrt{3} + i)$   
 d)  $x_1 = 1 + i, x_2 = 1 - i, x_3 = -1 + i, x_4 = -1 - i, x^4 + 4 = (x - 1 - i)(x - 1 + i)(x + 1 - i)(x + 1 + i)$   
 e)  $x_1 = \sqrt[4]{2} \left( \frac{1}{2} + \frac{\sqrt{3}}{2}i \right), x_2 = \sqrt[4]{2} \left( -\frac{\sqrt{3}}{2} + \frac{1}{2}i \right), x_3 = \sqrt[4]{2} \left( -\frac{1}{2} - \frac{\sqrt{3}}{2}i \right), x_4 = \sqrt[4]{2} \left( \frac{\sqrt{3}}{2} - \frac{1}{2}i \right),$   
 $x^4 + 1 + \sqrt{3}i = (x - x_1)(x - x_2)(x - x_3)(x - x_4)$

### 9. Feladat.

- a)  $f = g \cdot (x^3 + 2x^2 + 3x + 1) + 4$   
 b)  $f = g \cdot (x^3 + 3x^2 - 2) + x$   
 c)  $f = g \cdot (x^3 + \frac{1}{2}x + 2) - 2x - 1$

### 10. Feladat.

- a)  $\text{lnko} \{f, g\} = x + 1$   
 b)  $\text{lnko} \{f, g\} = x^2 + 6x - 7$   
 c)  $\text{lnko} \{f, g\} = x^4 - x^3 + x^2 - 2x + 1$