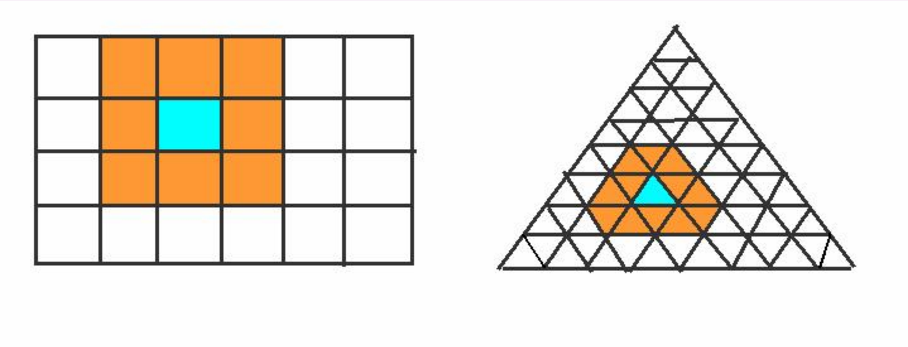
Szigetek maximális száma

K. Horváth Eszter, Szeged

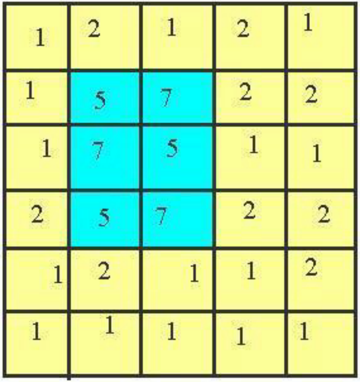
Miskolc, 2009. május 13.

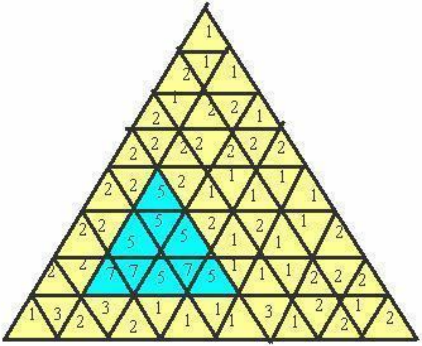
Definition/1

Grid, neighbourhood relation



We call a rectangle/triangle an *island*, if for the cell t, if we denote its height by a_t, then for each cell $\hat{t}$ neighbouring with a cell of the rectange/triangle T, the inequality $a_{\hat{t}} < min\{a_t : t \in T\}$ holds.





History/1

Coding theory

S. Földes and N. M. Singhi: On instantaneous codes, J. of Combinatorics, Information and System Sci., 31 (2006), 317-326.

History/1

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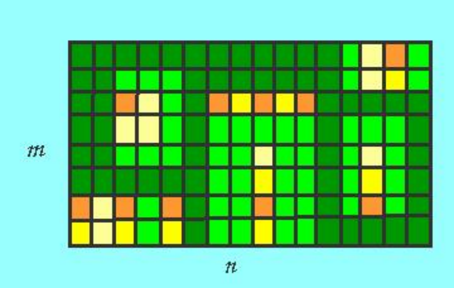
History/2

Rectangular islands

G. Czédli: The number of rectangular islands by means of distributive lattices, European Journal of Combinatorics 30 (2009), 208-215.

The maximum number of rectangular islands in a $m \times n$ rectangular board on square grid:

$$f(m,n) = [\frac{mn+m+n-1}{2}].$$



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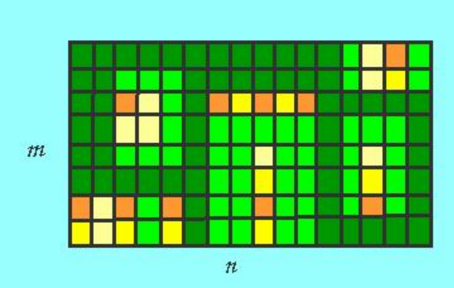

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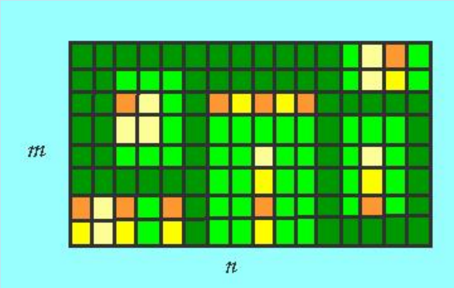

History/3

Rectangular islands in higher dimensions

G. Pluhár: The number of brick islands by means of distributive lattices, Acta Sci. Math., to appear.

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History/4

Further results on rectangular islands

Zs. Lengvárszky: The minimum cardinality of maximal systems of rectangular islands, European Journal of Combinatorics, available online 16 April 2008.

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Proving methods

J. Barát, P. Hajnal, E. K. Horváth: Elementary proof techniques for the maximum number of islands, earlier title: Islands, lattices and trees, submitted to the European Journal of Combinatorics.

Proving methods/1

LATTICE THEORETICAL METHOD

G. Czédli, A. P. Huhn and E. T. Schmidt: Weakly independent subsets in lattices, Algebra Universalis 20 (1985), 194-196.

Any two weak bases of a finite distributive lattice have the same number of elements.

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Proving methods/2

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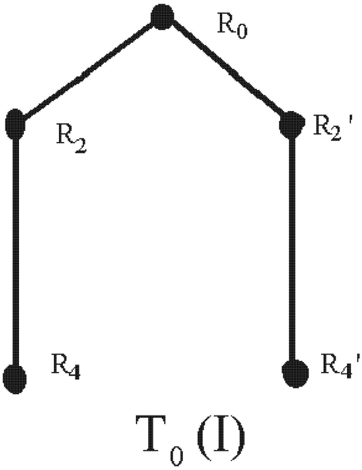

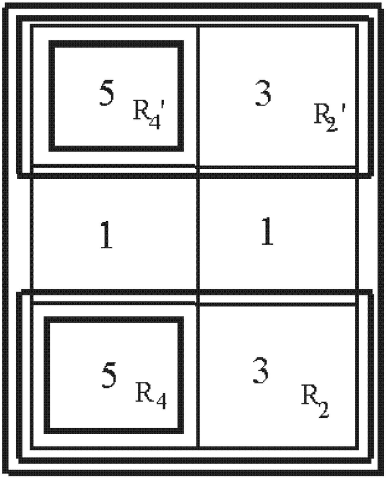

Proving methods/2

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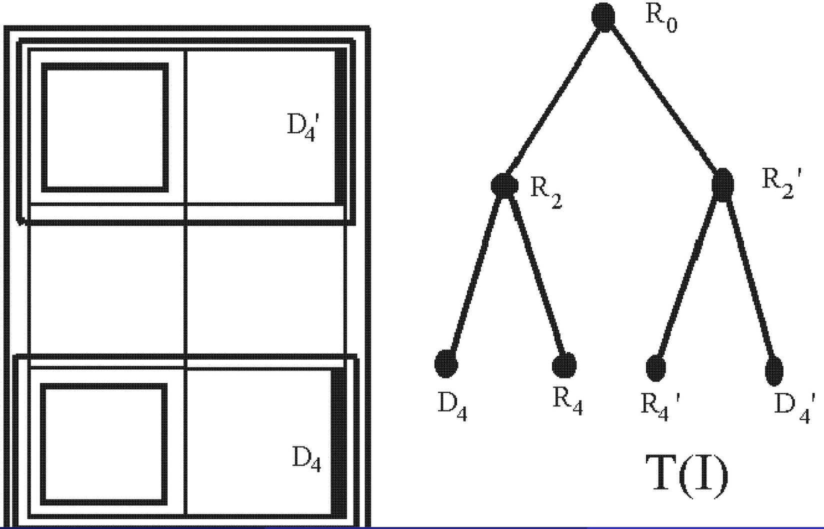

Proving methods/2

TREE-GRAPH METHOD

Lemma 2 (folklore)

(i) Let T be a binary tree with ℓ leaves. Then the number of vertices of T depends only on ℓ, moreover $|V| = 2\ell - 1$.

(ii) Let T be a rooted tree such that any non-leaf node has at least 2 sons. Let ℓ be the number of leaves in T. Then $|V| \leq 2\ell - 1$.

We have $4s + 2d \leq (n+1)(m+1)$.
The number of leaves of $T(\mathcal{I})$ is $\ell = s + d$. Hence by Lemma 2 the number of islands is

$$|V| - d \leq (2\ell - 1) - d = 2s + d - 1 \leq \frac{1}{2}(n+1)(m+1) - 1.$$

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Proving methods/3

ELEMENTARY METHOD

We define

$$\mu(R) = \mu(u, v) := (u + 1)(v + 1).$$

Now

$$f(m, n) = 1 + \sum_{R \in max\mathcal{I}} f(R) = 1 + \sum_{R \in max\mathcal{I}} ([\frac{(u+1)(v+1)}{2}] - 1)$$

$$= 1 + \sum_{R \in max\mathcal{I}} ([\frac{\mu(u,v)}{2}] - 1) \leq 1 - |max\mathcal{I}| + [\frac{\mu(\mathrm{C})}{2}].$$

If $|max\mathcal{I}| \geq 2$, then the proof is ready. Case $|max\mathcal{I}| = 1$ is an easy exercise.

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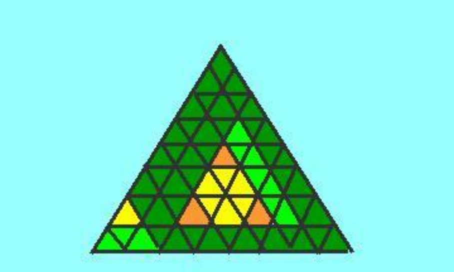
Estimating the maximum number of islands/1

Triangular islands

E. K. Horváth, Z. Németh and G. Pluhár: The number of triangular islands on a triangular grid, Periodica Mathematica Hungarica, 58 (2009), 25–34.
Available at http://www.math.u-szeged.hu/~horvath

For the maximum number of triangular islands in an equilateral rectangle of side length n, $\frac{n^2+3n}{5} \leq f(n) \leq \frac{3n^2+9n+2}{14}$ holds.



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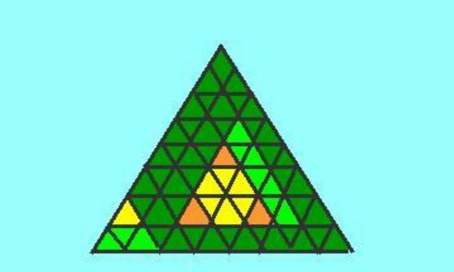

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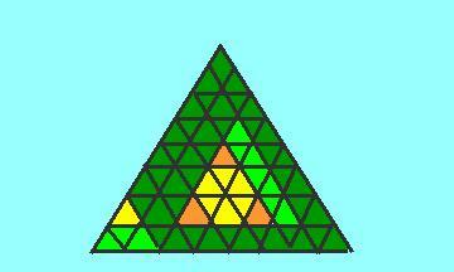
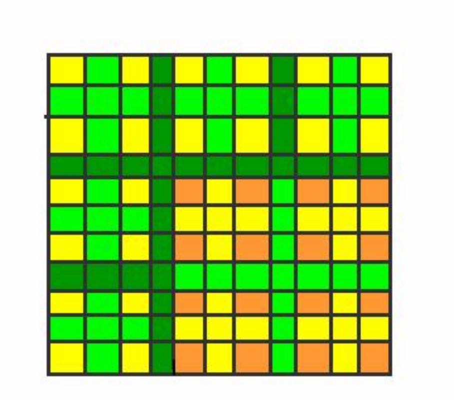


Estimating the maximum number of islands/2

Square islands (also in higher dimensions)

E. K. Horváth, G. Horváth, Z. Németh, Cs. Szabó: The number of square islands on a rectangular sea, Acta Sci. Math., submitted. Available at http://www.math.u-szeged.hu/~horvath



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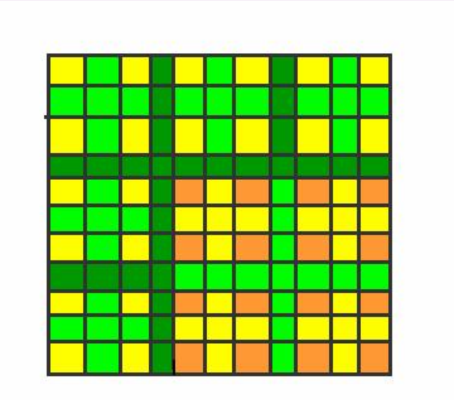

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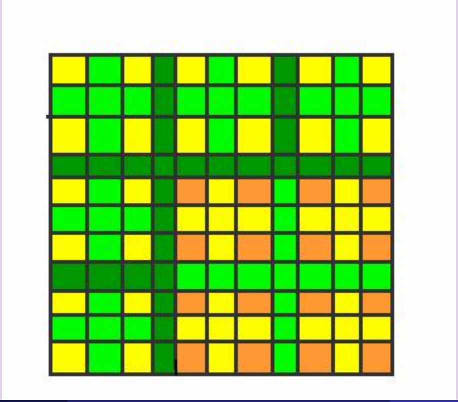

Exact results/1

Some exact formulas

Peninsulas (semi islands) (J. Barát, P. Hajnal, E.K. Horváth):
$p(m,n) = f(m,n) = [(mn + m + n - 1)/2]$.

Cylindric board, rectangular islands (J. Barát, P. Hajnal, E.K. Horváth):
If $n \geq 2$, then $h_1(m,n) = [\frac{(m+1)n}{2}]$.

Cylindric board, cylindric and rectangular islands (J. Barát, P. Hajnal, E.K. Horváth):
If $n \geq 2$, then $h_2(m,n) = [\frac{(m+1)n}{2}] + [\frac{(m-1)}{2}]$.

Torus board, rectangular islands (J. Barát, P. Hajnal, E.K. Horváth):
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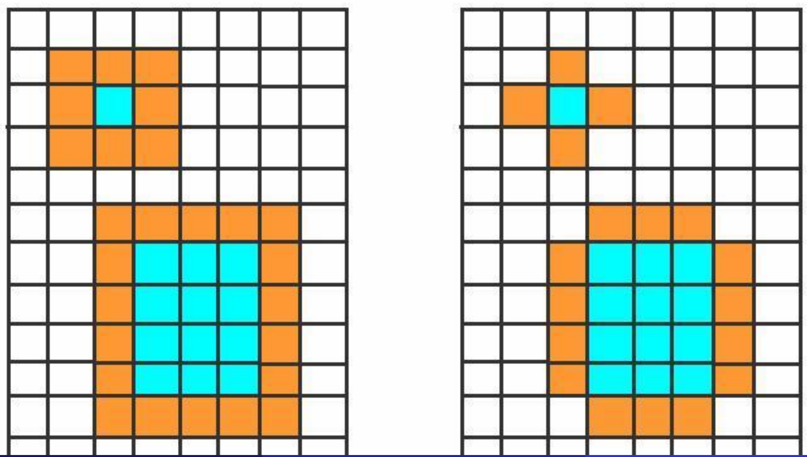
Several surfaces

Torus, globe

Exact results/2

Changing the neigbourhood relation of cells (J. Barát, P. Hajnal, E.K. Horváth): $f^*(m,n) = f(m,n) = [(mn + m + n - 1)/2]$.



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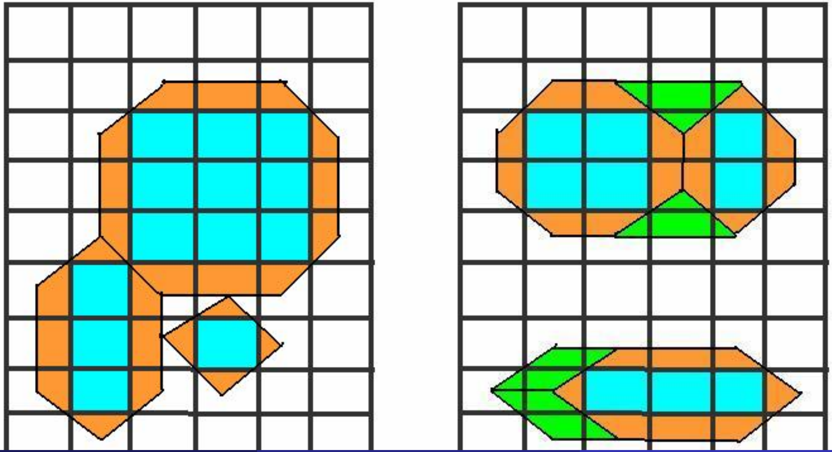

Islands in Boolean algebras, i.e. in hypercubes /1

Joint work with J. Barát and Péter Hajnal

The board consists of all vertices of a hypercube, i.e. the elements of a Boolean algebra $BA = \{0,1\}^n$.

We consider two cells neighbouring if their Hamming distance is 1.

We denote the maximum number of islands in $BA = \{0,1\}^n$ by $b(n)$.

Theorem 1

$b(n) = 1 + 2^{n-1}$.

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Islands in Boolean algebras, i.e. in hypercubes /2

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Proof:

$b(n) \geq 1 + 2^{n-1}$ because we can put one-cell islands to all vertices with an odd number of 1-s.

We show $b(n) \leq 1 + 2^{n-1}$ by induction on n. For $n = 0, 1$ the statement is easy to check.
For $n \geq 2$, we cut the hypercube into two half-hypercubes, of size 2^{n-1}. If one of them is an island, then the other cannot contain island.
If neither of them is an island, then by the induction hypothesis, in both half-hypercubes, the maximum cardinality of a system of islands is at most 2^{n-2}.

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Rectangular fuzzy relations/1

Joint work with Branimir Šešelja and Andreja Tepavčević

Let A and B nonempty sets and L a lattice. Then a *fuzzy relation* ρ is a mapping from $A \times B$ to L.

For every $p \in L$, cut relation is an ordinary relation ρ_p on $A \times B$ defined by

$$(x, y) \in \rho_p \text{ if and only if } \rho(x, y) \geq p.$$

We consider special lattice valued fuzzy relations:
The set $\{1, 2, ..., m\} \times \{1, 2, ..., n\}$, $m, n \in \mathbb{N}$, is called *a* table of size $m \times n$. Such a table is the domain of a fuzzy relation Γ:

$$\Gamma : \{1, 2, ..., m\} \times \{1, 2, ..., n\} \rightarrow \mathbb{N}.$$

The co-domain is the lattice $(\mathbb{N}, \leq)$, where $\mathbb{N}$ is the set of natural numbers under the usual ordering $\leq$ and suprema and infima are max and min, respectively.

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The co-domain is the lattice $(\mathbb{N}, \leq)$, where $\mathbb{N}$ is the set of natural numbers under the usual ordering $\leq$ and suprema and infima are max and min, respectively.

Rectangular fuzzy relations/1

Joint work with Branimir Šešelja and Andreja Tepavčević

Let A and B nonempty sets and L a lattice. Then a *fuzzy relation* ρ is a mapping from $A \times B$ to L.

For every $p \in L$, cut relation is an ordinary relation ρ_p on $A \times B$ defined by

$$(x, y) \in \rho_p \text{ if and only if } \rho(x, y) \geq p.$$

We consider special lattice valued fuzzy relations:
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Rectangular fuzzy relations/2

We say that two rectangles $\{\alpha, ..., \beta\} \times \{\gamma, ..., \delta\}$ and $\{\alpha_1, ..., \beta_1\} \times \{\gamma_1, ..., \delta_1\}$ are *distant* if they are disjoint and for every two cells, namely (a, b) from the first rectangle and (c, d) from the second, we have $(a - c)^2 + (b - d)^2 \geq 4$.

Fuzzy relation Γ is called *rectangular* if for every $p \in \mathbb{N}$, every nonempty p-cut of Γ is a union of distant rectangles.

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Rectangular fuzzy relations/3

5	5	3	5	5
4	4	2	4	4
2	2	1	2	2

$S_1 = \{1, 2, 3, 4, 5\} \times \{1, 2, 3\}$,
$S_2 = \{1, 2, 3, 4, 5\} \times \{1, 2, 3\} \setminus \{(3, 1)\}$,
$S_3 = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 3), (4, 2), (4, 3), (5, 2), (5, 3)\}$,
$S_4 = \{(1, 2), (1, 3), (2, 2), (2, 3), (4, 2), (4, 3), (5, 2), (5, 3)\}$ and
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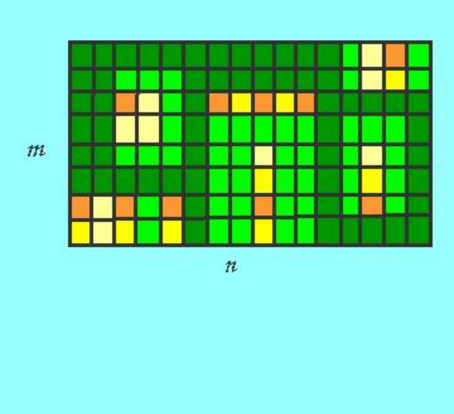
For every fuzzy relation $\Gamma : \{1, 2, ..., n\} \times \{1, 2, ..., m\} \to \mathbb{N}$, there is a rectangular fuzzy relation $\Phi : \{1, 2, ..., n\} \times \{1, 2, ..., m\} \to \mathbb{N}$, having the same islands.



Rectangular fuzzy relations/4

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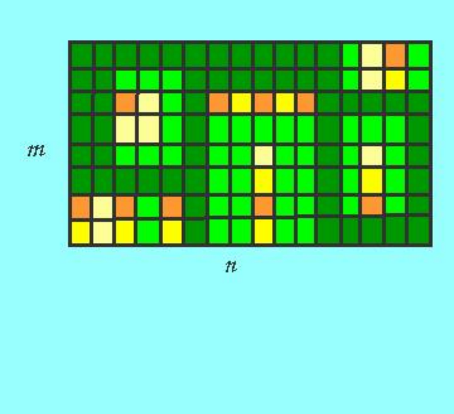

Theorem 3
For every rectangular fuzzy relation $\Phi : \{1, 2, ..., n\} \times \{1, 2, ..., m\} \to \mathbb{N}$, there is a rectangular fuzzy relation $\Psi : \{1, 2, ..., n\} \times \{1, 2, ..., m\} \to \mathbb{N}$, having the same islands and in Ψ every island appears exactly in one cut.

If a fuzzy rectangular relation Ψ has the property that each island appears exactly in one cut, then we call it *standard fuzzy rectangular relation.* We denote by $\Lambda(m, n)$ the maximum number of different p-cuts of a standard fuzzy rectangular relation on the rectangular table of size $m \times n$.

Theorem 4
$\Lambda(m, n) = m + n - 1.$

Rectangular fuzzy relations/5

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Rectangular fuzzy relations/6

We have further results, e.g.
Characterization Theorem for rectangular fuzzy relations,
Constructing Algorithm which constructs rectangular fuzzy relation for a given arbitrary fuzzy relation with the same islands.

E. K. Horváth, Branimir Šešelja and Andreja Tepavčević: Cut approach to islands in rectangular fuzzy relations, submitted to Fuzzy Sets and Systems.

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Islands for high school students

E. K. Horváth, Andreja Tepavčević, A. Máder: Introducing Czédli-type islands, submitted to The Mathematical Gazette.

Determine the maximum number of islands in the one-dimensional board of length n, if the integers in the cells are elements of the set $\{0, 1, 2, \ldots, h\}$. We suppose that the board is bordered with cells filled with 0 at the ends, i.e. the 0th and the $n+1$-th cells are filled with 0. Therefore, in case there is no any 0 appeared in the board, the board itself is considered to be an island.

The maximum number of islands: $I(n, h) = n - \left\lfloor \frac{n}{2^h} \right\rfloor$, where $\lfloor x \rfloor$ denotes the lower integer part of x.

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