Maximal regular simplicial complexes

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Call a set $S \subset \Re^n$ *regular* iff there exists $a > 0$ such that whenever $\{x, y\} \subseteq S$ then $|x - y| \in \{0, a\}$, where $|x - y|$ is the euclidean distance between x and y.

Of course a maximal regular subset of $\Re^2$ has exactly three elements. Furthermore, an equilateral triangle of edge length a has height $h_2 = a\sqrt{3}/2$ and circumcircle diameter $d_2 = 2a/\sqrt{3}$. Note that $h_2 d_2 = a^2$.

A maximal regular subset of $\Re^3$ has exactly four elements. Moreover, we easily compute that the height of a regular tetrahedron of edge length a is $h_3 = a\sqrt{2/3}$ and that its circumsphere's diameter is $d_3 = a\sqrt{3/2}$. So $h_3 d_3 = a^2$.

The ideas suggested by the foregoing two paragraphs hold also in $\Re^1$: That is, two is the size of a maximal regular subset of $\Re$, and $h_1 d_1 = a^2$ in the case of a line segment of length a. (The figure is a line segment, and its "circumcircle" consists of the two endpoints of the interval.)

We establish the natural generalization of those ideas to $\Re^n$.