EXTENT PARTITIONS AND CONTEXT EXTENSIONS

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Abstract

An extent partition of a formal context $\mathbb{K} = (G, M, I)$ is a partition $\pi = \{G_t \mid t \in T\}$ of G, all classes of which are concept extents, that is $G_t'' = G_t$, for all $t \in T$. Note that, since the intersection of extents always yields an extent, the common refinements of extent partitions are again extent partitions. Therefore the extent partitions of $\mathbb{K}$ form a complete $\wedge$-subsemilattice of the partition lattice of G, and thus a complete lattice. In particular, there is always a finest extent partition of (G, M, I) which we denote by $\pi_\square$.

We call a set $E \subseteq G$ a *box extent* of the context $\mathbb{K}$, if E is a class of some extent partition of $\mathbb{K}$, or $E = \emptyset''$. The intersection of box extents is a box extent, moreover, an extent $E \neq \emptyset''$ is a box extent iff it is a union of some classes of $\pi_\square$. In other words E is a box extent iff

$$g \in E \Rightarrow g^{\square\square} \subseteq E.$$

As a consequence, the box extents form a closure system and therefore a complete lattice $\mathbf{Box}(\mathbb{K})$. We prove that $\mathbf{Box}(\mathbb{K})$ is a complete atomistic lattice, where the atomic extents are exactly the classes of the finest extent partition $\pi_\square$ of $\mathbb{K}$. We also characterise $\mathbf{Box}(\mathbb{K})$ as the largest atomistic complete $\wedge$-subsemilattice of the lattice of the concept extents of $\mathbb{K}$.

$\mathbb{K}$ is called a *one-object extension* of the subcontext (H, M, J) if it is obtained by adding a new element with attributes in M to the set H. We investigate the interplay between the box extents of (H, M, J) and those of its one-object extension $\mathbb{K}$, and describe those box extents of (H, M, J) which can be extended to $\mathbb{K}$.