THE VISUAL STRUCTURE OF PLANAR SEMIMODULAR LATTICES AND AN APPLICATION

GÁBOR CZÉDLI AND E. TAMÁS SCHMIDT

A finite lattice L is called *slim* if no three join-irreducible elements of L form an antichain. Slim lattices are planar. So, they are relatively easy objects to understand.

Because of their links to combinatorics and geometry, the study of (upper) *semimodular lattices* is an important branch of Lattice Theory. Moreover, slim semimodular lattices have recently proved to be useful in strengthening a classical group theoretical result, namely, the Jordan-Hölder theorem.

After exploring some elementary properties of slim lattices and slim semimodular lattices, we give two visual *structure theorems* (in fact, twin theorems) for slim semimodular lattices. The first theorem says that each slim semimodular lattice L can be obtained from a chain by using the following two operations: (i) adding a fork, and (ii) adding a corner. His "twin brother" asserts that L can also be obtained from the direct product of two nontrivial finite chains such that (i) first we add finitely many forks one by one, and then (ii) we remove corners, one by one.

As one of the applications of this machinery, the connection to the Jordan-Hölder theorem is mentioned.

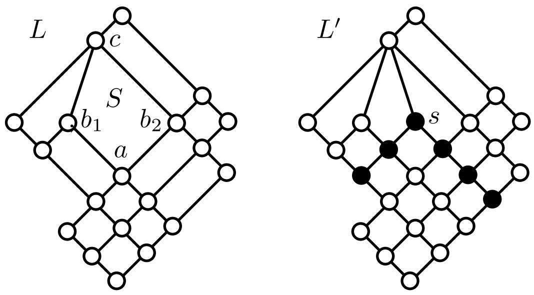

FIGURE 1. Adding a fork to L

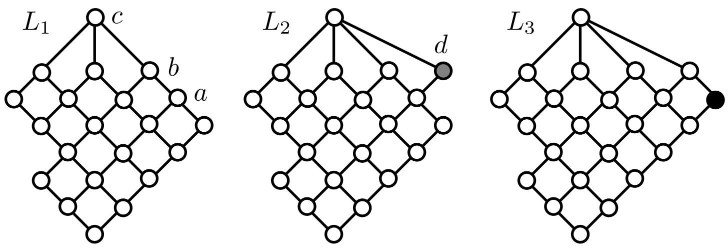

FIGURE 2. Weak corner and corner