

Superimposed codes

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There are many instances in Coding Theory when codewords must be restored from partial information, like defected data (error correcting codes), or some superposition of the strings. These lead to superimposed codes, a close relative of group testing problems.

There are lots of versions and related problems, like Sidon sets, sum-free sets, union-free families, locally thin families, cover-free codes and families, etc. We discuss two cases *cancellative* and *union-free* codes.

A family of sets \mathcal{F} (and the corresponding code of 0-1 vectors) is called **union-free** if $A \cup B \neq C \cup D$ and $A, B, C, D \in \mathcal{F}$ imply $\{A, B\} = \{C, D\}$. \mathcal{F} is called *t-cancellative* if for all distinct $t + 2$ members A_1, \dots, A_t and $B, C \in \mathcal{F}$

$$A_1 \cup \dots \cup A_t \cup B \neq A_1 \cup \dots \cup A_t \cup C.$$

Let $c_t(n)$ be the size of the largest *t-cancellative* code on n elements. We significantly improve the previous upper bounds of Körner and Sinaimeri, e.g., we show $c_2(n) \leq 2^{0.322n}$ (for $n > n_0$). We introduce a method to deal with such problems, namely we first we investigate the constant weight case (i.e., uniform hypergraphs).