

## The mathematics of G. Grätzer and E. T. Schmidt

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I think it is hopeless to try to give a concise account of the over 300 publications of George Grätzer and E. Tamás Schmidt. There is a danger of getting lost in too many details (for instance, mentioning Grätzer [G 35], where the concept of Mal'cev condition was introduced and named). Hence, I will be guided by the following five restrictions. (1) I will discuss only lattice theoretic results. (2) I will try to concentrate on their deepest results, (3) on results with the largest impact, and (4) on series of papers. (5) I will give preference to joint or at least partially joint works. Even then, I will be able to cover only a small part of their results satisfying these five criteria.

### 1. Characterizing congruence lattices of algebras

In [GS 19], when they were only 26 years of age, Grätzer and Schmidt proved their best known result (the so-called Grätzer–Schmidt Theorem), solving a famous open problem of G. Birkhoff.

**Theorem.** *Congruence lattices of (finitary) algebras are characterized as algebraic lattices.*

About half a dozen new proofs have been published in the 45 years after the publication in [GS 19]. No one has found a magical two page proof of this result.

### 2. Congruence lattices of finite lattices

The papers [GS 19, 32, 33, 40–42, 46–48, 51, 54, 56, 57] and [G 155] all deal with representing a finite distributive lattice as the congruence lattice of a finite lattice with special properties. For some closely related results, see Schmidt [S 14, 16, 28]. The topic is surveyed in Grätzer's new book ([GB 7]). A typical result is the following (Grätzer, Lakser, and Schmidt [GS 40]):

**Theorem.** *Every finite distributive lattice can be represented as the congruence lattice of a finite semimodular lattice.*

For many representation results, there are “congruence-preserving extension” variants, which are usually much harder to prove. Here is an example ([GS 47]):

**Theorem.** *Every finite lattice has a congruence-preserving extension to a finite semimodular lattice.*

In an interesting combinatorial aspect of this field, one inquires about the minimum size of the lattice  $L$  that represents a distributive lattice  $D$  with  $n$  join-irreducible elements. Many results give upper bounds for the size of  $L$ , but only for two cases has it been proved that the upper bound is optimal.

**Theorem.** *Every finite distributive lattice  $D$  can be represented as the congruence lattice of a lattice  $L$  of size  $O(n^2)$  and the size  $O(n^2)$  is optimal.*

The size  $O(n^2)$  construction is due to Grätzer, Lakser, and Schmidt [GS 32], and the optimal part to Grätzer, Rival, and Zaguia [G 122]. There is only one other upper bound, by Grätzer and E. Knapp [G 155], for which there is an optimal part by the same authors (submitted). The most complicated paper on the subject is Grätzer and Lakser (*Representing homomorphisms of congruence lattices as restrictions of congruences of isoform lattices*, submitted to Acta Sci. Math. (Szeged) in Nov. 2007), which builds on [GS 51, 55, 57] and [G 121].

### 3. Congruence lattices of lattices

It has long been conjectured that congruence lattices of lattices can be characterized as distributive algebraic lattices; equivalently, as ideal lattices of distributive join-semilattices with zero. One of the deep positive results is due to Schmidt [S 7] and [S 25]:

**Theorem.** *Let  $L$  be the ideal lattice of a distributive lattice with zero. Then  $L$  can be represented as the congruence lattice of a lattice.*

For a detailed accounting of this field, see [G 151], which lists all the positive results and the surprising counterexample to this conjecture by F. Wehrung.<sup>1</sup>

### 4. Complete congruence lattices of complete lattices

G. Birkhoff asked in 1945 what are the congruence lattices of infinitary algebras. This problem was solved by G. Grätzer and W. A. Lampe in Appendix 7 of [GB 4]. Answering an even more difficult question, Grätzer [GP 7] proved that

**Theorem.** *Complete congruence lattices of complete lattices are characterized as complete lattices.*

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<sup>1</sup>A solution of Dilworth's congruence lattice problem, Adv. Math. **216** (2007), 610–625

This was followed up in a series of papers [GS 21, 23–26, 28–30, 38, 51]. The best result is in [GS 29]:

**Theorem.** *Complete congruence lattices of complete distributive lattices are characterized as complete lattices.*

## 5. Independence results

Grätzer [GB 6] raised the following problem (Problem II.18):

*Let  $L$  be a nontrivial lattice and let  $G$  be a group. Does there exist a lattice  $K$  such that  $K$  and  $L$  have isomorphic congruence lattices and the automorphism group of  $K$  is isomorphic to  $G$ ?*

The finite case was solved in the affirmative, independently in 1978 by V.A. Baranskii and A. Urquhart. In 1995 Grätzer and Schmidt [GS 30] proved a much stronger result, the *strong independence* of the automorphism group and the congruence lattice in the finite case. In [G 134] (a joint paper with F. Wehrung), this was generalized to arbitrary lattices. (And the proof jumps from 5 to about 50 pages, if we include the relevant portions of [G 127]!)

**Theorem** (The Strong Independence Theorem for Lattices).

*Let  $L_A$  and  $L_C$  be lattices, and let  $L_C$  have more than one element. Then there exists a lattice  $K$  that is an automorphism-preserving extension of  $L_A$  and a congruence-preserving extension of  $L_C$ . If  $L_A$  and  $L_C$  are countable, then  $K$  can be constructed as a countable lattice.*

## 6. Extension theorems

In Schmidt [S 14, 16], a very important construction was introduced. Let  $D$  be a bounded distributive lattice. Then there exists a modular lattice  $M_3[D]$  that contains a  $\{0,1\}$ -sublattice  $M_3 = \{o, a, b, c, i\}$  with  $[0, a] \cong D$ . This had many applications, for instance, [S 14, 16] and [GS 53]. The  $M_3[D]$  construction was generalized by Grätzer and Wehrung [G 126] by dropping the requirement that  $D$  be distributive. This led to a solution of a long-standing problem:

**Theorem.** *Every lattice has a proper congruence-preserving extension.*

This was further generalized by Grätzer and Wehrung to arbitrary lattices, see [G127, 129, 133], and applied in [G 134]. For a survey of this field, see [G 136]. Another direction is due to Grätzer and M. Greenberg, see [G 138–140, 142].

## 7. Freely adjoining a (relative) complement to a lattice

This field starts with a celebrated result of R. P. Dilworth (1944): *Every lattice can be embedded into a uniquely complemented lattice.* In 1969 C. C. Chen and Grätzer [G 29] found a stronger form of this result:

**Theorem.** *Every bounded at most uniquely complemented lattice has a  $\{0, 1\}$ -embedding into a uniquely complemented lattice.*

The proof utilizes an analysis of the free lattice obtained by freely adjoining a complemented element to a lattice, and this provides a much simplified proof of Dilworth's result. The technique introduced there has found many applications; see Grätzer [G 34, 45, 86, 93, 143, 148] and [GP 3]. Here is a sample result from Grätzer and H. Lakser [G 143]:

**Theorem.** *Let  $K$  be a lattice, and let  $[a, c]$  be an interval in  $K$ . If  $[a, c]$  in  $K$  is at most uniquely relatively complemented, then  $K$  has an extension  $L$  such that the interval  $[a, c]$  of  $L$  is uniquely relatively complemented.*

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- [GS 4] G. Grätzer and E. T. Schmidt, *Ideals and congruence relations in lattices. I*, Magyar Tud. Akad. Mat. Fiz. Oszt. Közl. **7** (1957), 93–109 (Hungarian).
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## Schmidt minus Grätzer

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