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Magari via Malcev

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A variety $\mathscr{V}$ is called *semidegenerate*, if the non-trivial algebras of $\mathscr{V}$ have no idempotent elements (i.e. one-element subalgebras; see [2]). This is a Malcev property: a variety $\mathscr{V}$ is semidegenerate iff, for some n, there are ternary terms $t_1, \dots, t_n$ and unary terms $u_1, \dots, u_{2n}$ such that

$$\begin{aligned} &t_1(u_1(x), x, y) = x, \\ &t_i(u_{2i}(x), x, y) = t_{i+1}(u_{2i+1}(x), x, y) \qquad (i = 1, \dots, n-1) \qquad (*) \\ &t_n(u_{2n}(x), x, y) = y \end{aligned}$$

hold identically in $\mathscr{V}$ (see [1]).

Using this notion, we give a symmetric proof for the theorem of Magari on the existence of simple algebras in varieties [3]. In addition, this proof makes it apparent that simple algebras are omnipresent in varieties. Here it follows:

Let $\mathscr{V}$ be a non-trivial variety. Suppose $\mathscr{V}$ is

not semidegenerate.	semidegenerate.

Let $\mathbf{A}$ $(\in \mathscr{V})$ be generated by

$\{a, b\}$, where a is idempotent. Then $Cg^{\mathbf{A}}(a, b) =$	one element c. By $(*)$, $Cg^{\mathbf{A}}(u_1(c), \dots, u_{2n}(c)) =$

A^2.

Hence, any $\lambda \in Con\,\mathbf{A}$ which is maximal among those separate (at least two of)

a and b	$u_1, \dots, u_{2n}$

– such a λ exists by Zorn – is a maximal proper congruence of $\mathbf{A}$. Thus, $\mathbf{A}/\lambda$ $(\in \mathscr{V})$ is simple.

Presented by R. W. Quackenbush.
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