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Sets of minimal capacity

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Let f be an analytic function element at $\infty \in \overline{\mathbb{C}}$, and $D \subset \overline{\mathbb{C}}$, $\infty \in D$, a domain into which the function f can be extended analytically (or meromorphically) in a single-valued manner. Among all such domains D, we look for a domain D_0 such that $\mathrm{cap}(D_0)$ is **minimal**.

The (uniquely existing) domain D_0 is called **extremal domain** and its complement $K_0 := \mathbb{C} \setminus \mathbb{D}_0$ **minimal set**. Such domains and sets play an important role in rational approximation, and related definitions have also significant relevance in other areas of application. Traditionally, the home territory of this type of problems and also of the tools for their analysis is the geometric theory of functions.

We shall review several variants of definition, and will thereby emphasise the potential-theoretic aspects of the problem. Some of the definitions are equivalent, others only similar, while the differences between them are intresting and fruitful topics for discussion. Further, we will describe the structure of minimal sets K_0, and will discuss the key properties of a function f that are determinants for its associated minimal set K_0.

In the world of geometric function theory, sets of minimal capacity are closely connected with the concept of quadratic differentials and their trajectories. So-called critical trajectories turn out to be building blocks for the minimal sets. We intend to touch also this aspect, if time permits, and will discuss its connection with a symmetry property of the Green function of the extremal domain D_0.

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