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On the generic behaviour of Padé approximants to functions with 3 branch points

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Given non-collinear points a_1 , a_2 , a_3 , there is a unique compact $\Delta \subset \mathbf{C}$, called Chebotarëv continuum, of minimal logarithmic capacity among all continua joining a_1 , a_2 , and a_3 . It consists of three analytic arcs Δ_k , $k \in$ $\{1, 2, 3\}$, that emanate from a common endpoint, say a_0 , and is the closure of the negative trajectories of the quadratic differential $((z - a_0)/\Pi_{k=1}^3(z - a_k))dz^2$. For h be a complex-valued non-vanishing Dini-continuous function on Δ , we consider the function

$$f_h(z) := \frac{1}{\pi i} \int_{\Delta} \frac{h(t)}{t - z} \frac{dt}{w^+(t)}$$

where $w(z) := \sqrt{\prod_{k=0}^{3} (z - a_k)}$ and w^+ is the one-sided value according to some orientation of Δ . We investigate the domain of uniform convergence of Padé approximants to f_h and and prove it is generically empty, in the measure-theoretic sense, with respect to the location of the a_k . Still, some subsequence does converge generically. We also show that non-empty domain densely occurs. We rely in part on former work by Kuzmina and Suetin. This is joint work with M. Yattselev.