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On the generic behaviour of Padé approximants to functions with 3 branch points

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Given non-collinear points a_1, a_2, a_3, there is a unique compact $\Delta \subset \mathbf{C}$, called Chebotarëv continuum, of minimal logarithmic capacity among all continua joining a_1, a_2, and a_3. It consists of three analytic arcs Δ_k, $k \in \{1,2,3\}$, that emanate from a common endpoint, say a_0, and is the closure of the negative trajectories of the quadratic differential $((z-a_0)/\Pi_{k=1}^3(z-a_k))dz^2$. For h be a complex-valued non-vanishing Dini-continuous function on Δ, we consider the function

$$f_h(z) := \frac{1}{\pi i}\int_\Delta \frac{h(t)}{t-z}\frac{dt}{w^+(t)},$$

where $w(z) := \sqrt{\prod_{k=0}^3 (z-a_k)}$ and w^+ is the one-sided value according to some orientation of Δ. We investigate the domain of uniform convergence of Padé approximants to f_h and and prove it is generically empty, in the measure-theoretic sense, with respect to the location of the a_k. Still, some subsequence does converge generically. We also show that non-empty domain densely occurs. We rely in part on former work by Kuzmina and Suetin. This is joint work with M. Yattselev.