Szeged Dynamics Days

Szeged, 28 February – 1 March

List of abstracts

Talks

Monotonicity of critical points on planar curves evolving under curvature-driven flows

Gábor Domokos
Budapest University of Technology and Economics

We consider the shrinking of a smooth, closed convex planar curve with inward normal speed v given as $v = v(\kappa)$ a function of the curvature κ. Grayson proved in 1987 that if $v = \kappa$ then the number $N(t)$ of spatial critical points (extremal points of a polar distance function measured from a fixed origin) is monotonically decreasing. Here we generalize this result by showing that for the monotonic decrease of $N(t)$ it is sufficient that $\frac{\partial v}{\partial \kappa} > 0$.

Connection between the linear and nonlinear stability notions in numerical analysis

Petra Csomós, István Faragó, Imre Fekete
Institute of Mathematics, Eötvös Loránd University and MTA–ELTE Research Group "Numerical analysis and large networks"

In the present talk we analyse the nonlinear stability notions in the linear case. We assume that the semidiscretised problem can be formulated as an abstract inital value problem on some Banach space. Its solution is then obtained with the help of a linear one-parameter operator semigroup which can be further approximated by using certain time discretisation methods. We will show how the usual stability conditions applied in the linear case imply the nonlinear-type stability. The results are illustrated by a simple example as well.

Interconnection between ordinary and delay differential equations via approximation techniques

István Győri
University of Pannonia, Veszprém

Our lecture is focussing on two approximation techniques and their applications in mathematical biology. In both cases, we establish connections between the solutions of a delay differential equation and of a suitably constructed n dimensional system of ordinary differential equations. The two approximating schemes are originated from two different ideas. The first one is based on the connection between transit compartments models which is frequently used in pharmacology and the theory of compartmental models with pipes. The second scheme is derived from a relation between delay differential equations and hyperbolic type partial differential equations which is combined with the method of lines approximation.

In both cases the convergence is uniform on any compact interval as the dimension n tends to infinity. It is worth to note that in the first approximation case the convergence is also uniform on $[0, \infty)$, under some extra condition.

For background articles see the references below.

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On neutral differential equations with state-dependent delays

Ferenc Hartung
University of Pannonia, Veszprém

In this talk we consider a class of neutral differential equation with state-dependent delays in both in the neutral and the retarded terms. We discuss the well-posedness and continuous dependence issues and differentiability of the solutions with respect to parameters.

A conjecture connected with the stabilization of the inverted pendulum

László Hatvani
Bolyai Institute, University of Szeged

It was a surprising discovery that the unstable upper equilibrium of the mathematical pendulum can be stabilized by vibrating of the point of suspension vertically with sufficiently high frequency. By the use of Floquet theory it can be proved that the characteristic multipliers of the *linearization* of the equation of motion (i.e., the eigenvalues of the monodromia matrix of the linearized equation) are located on the unit circle on the complex plane, so the upper equilibrium state is stable, provided that the frequency of the excitation is large and the amplitude is small enough. The problem is that the monodromia matrix is not hyperbolic, therefore the stability of the equilibrium of the linearized system does not imply that of the equilibrium of the original *nonlinear system*. Results of computer experiments inspired us to formulate a conjecture about a property of trajectories of the original *nonlinear system*, which would be stronger than the stability of the equilibrium. This property reminds us of the center around the lower stable equilibrium of the classical pendulum, so we will call the equilibrium with this property a *non-autonomous center*.

Stability in Ambarzumian theorems

Miklós Horváth
Budapest University of Technology and Economics

The famous theorem of Ambarzumian states that if the eigenvalues of the problem $-y'' + q(x)y = \lambda y$ on $[0, \pi]$, $y'(0) = y'(\pi) = 0$ of the zero potential $q_0 = 0$ are the eigenvalues of the potential q then $q = 0$. We extend this statement to Schrödinger operators in arbitrary dimension: $-\Delta u + q(x)u = \lambda u$ on Ω, $\partial u/\partial \nu = 0$ on $\partial\Omega$. The main tool of the proof is the fact that given two potentials q and q^* the average shift of the potentials is equal to the average shift of the eigenvalues, that is,

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} (\lambda_k(q^*) - \lambda_k(q)) = \frac{1}{|\Omega|} \int_{\Omega} (q^* - q).$$

Finally we show that the stability of the recovery of the zero potential from the Neumann eigenvalues is almost optimal for sufficiently smooth domain and potential. In other words, if the average shift $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} (\lambda_k(q) - \lambda_k(0))$ is small and $\lambda_1(q) - \lambda_1(0)$ is small then $\|q\|_{L_2(\Omega)}$ is also small.

Positive invariance of sets and strong stability of numerical methods for time dependent differential equations

Zoltán Horváth
Széchenyi István University, Győr

Positively invariant sets (defined typically by inequalities on the solution) and forward in time monotone decreasing property of certain functionals of the solution are special features of many classes of initial (or initial-boundary) value problems for time dependent differential equations (ODEs, PDEs, DDEs). As typical examples when these properties are present we mention models for fluid flow, heat transfer, biological and chemical reactions and traffic flow. These properties, when present, are to be preserved by the numerical discretization process to have an efficient and physically reliable numerical method. However, this request often induces severe restrictions of the time step sizes of the discretization process.

In this talk we present our results on a formula for time step size threshold under which a numerical method preserves the suitable property of the continuous time problem under convexity assumptions on the positively invariant set and the monotone functional. Our analysis is valid for a wide class of numerical methods including Runge-Kutta, linear multistep, linearly implicit, peer and general linear methods. For a demonstration of our findings we show computational results for a nonlinear hyperbolic conservation law.

Sensory uncertainty and stick balancing at the fingertip

Tamás Insperger, John G. Milton
Budapest University of Technology and Economics

The effects of sensory input uncertainty on the stability of time-delayed human motor control are investigated by calculating the minimum stick length that can be stabilized in the inverted position for a given time delay. Five control strategies often discussed in the context of human motor control are examined: three time-invariant controllers: proportional-derivative (PD), proportional-derivative-acceleration (PDA), model predictive (MP) controllers, and two time-varying controllers: act-and-wait (AAW) and intermittent predictive (IP) controllers. The uncertainties of the sensory input are modeled as a multiplicative term in the system output. It is found that for an uncertainty of 7–13% a MP controller is the most robust controller. However, a PDA controller becomes more effective when the sensory uncertainty is larger than 15%.

On some nonlinear cross-diffusion systems

Róbert Kersner
University of Pécs

We consider a two-species competition system (“tumor system”) with nonlinear cross-diffusion and exhibit exact and numerical solutions of the system.

We first show the existence of spatially stationary solutions that are non constant but periodic patterns. In a particular case, we also provide a time-dependent solution that approximates this periodic solution. In the case of equal intrinsic growth rates, we give a sharp wavefront solution with semi-finite support.

Rich bifurcation structure in two variants of a classical vaccination model

Diána H. Knipl
MTA–SZTE Analysis and Stochastics Research Group, University of Szeged

Classical disease transmission models typically have a unique locally asymptotically stable steady state, the disesase free equilibrium, whenever a certain threshold known as the basic reproduction number ($\mathcal{R}_0$) is less than one. However, it is also possible to have a very different situation: when the model undergoes a backward bifurcation at $\mathcal{R}_0 = 1$, the DFE coexists with one stable positive and one unstable positive equilibrium for $\mathcal{R}_0 < 1$.
Such a behavior has been experienced in some simple SIVS (susceptible–infected–vaccinated–susceptible) disease transmission models. In this talk we consider an extension of this model by incorporating the possibility of immigration of susceptible and vaccinated individuals, and after showing global stability results we examine in detail how the structure of the bifurcation diagram depends on the immigration. The basic one-patch model is also generalized to the case when the population is divided into two regions which are connected by transportation. We give condition for the coexistence of multiple positive steady states and illustrate the rich dynamical behavior of the system with numerical simulations.
Joint work with Paweł Pilarczyk and Gergely Röst. DHK was supported by the European Union and the State of Hungary, co-financed by the European Social Fund in the framework of TÁMOP-4.2.4.A/ 2-11/1-2012-0001 'National Excellence Program'.

Understanding, treating, and avoiding human hematopoietic disease: Better medicine through mathematics?

Michael C. Mackey
McGill University, Montreal, Canada

In this talk I will outline the results of work we are conducting on the modeling of the regulation of human blood cell production and the uses we have put this modeling to in the understanding and treatment of hematopoietic diseases. In the last part I will discuss current work to minimize the hematopoietic side effects of chemotherapy. All of the models on which this work is based are framed as nonlinear differential delay equations, sometimes with state-dependent delays.

Transient behaviors in noisy and delayed bistable dynamical systems

John G. Milton
Claremont College, Claremont CA, USA

Do the observed behaviors of complex dynamical systems correspond to the asymptotic behaviors of proposed mathematical models (e.g. fixed-points, limit cycles, chaotic attractors) or to their transient behaviors? Although, it is known long duration transient behaviors can arise in time-delayed dynamical systems, such as micro-chaos, delayed-induced transient oscillations (DITOs), the effects of random perturbations ("noise") on their duration has received little attention. Here I discuss the effects of noise of the occurrence of DITOs in a two-neuron model with delayed mutual inhibition that arises in the context of the timing of seizures in a rare form of epilepsy (nocturnal frontal lobe epilepsy). Since the autocorrelation function decays rapidly it is possible to use a Markov chain approximation to gain insights into the duration of the DITOs.

Age-structured epidemic model with infection during transportation

Yukihiko Nakata
Bolyai Institute, University of Szeged

The increasing volume of international trade and tourism highly facilitates the rapid spread of infectious diseases around the world. We formulate an infection age-structured epidemic model, as a coupled system of renewal equations and delay differential equations, to describe spatial dynamics of infectious diseases that are transmitted not only at geographic locations but also during the transportation among regions (patches). I introduce a system of nonautonomous delay differential equations to construct modelling ingredients for delayed transition of individuals between two regions. Then I would like to show that an infinite-dimensional renewal equation arises as birth state of infectious individuals during the transportation is continuous with respect to the time elapsed since the start of travel. Joint work with D. H. Knipl and G. Röst.

Large time behavior of a linear delay differential equation with asymptotically small coefficient

Mihály Pituk
Department of Mathematics, University of Pannonia, Veszprém

In this talk we will consider the linear delay differential equation

$$x'(t) = p(t)x(t-r), \qquad t \geq t_0,$$

where $r > 0$ and $p\,[t_0, \infty) \to \mathbb{R}$ is a continuous function such that

$$\int_t^{t+r} |p(s)|\, ds \to 0, \qquad t \to \infty.$$

We will show that the large time behavior of the solutions can be described in terms of a special solution of the associated formal adjoint equation and the initial data. In the special case of the Dickman–de Bruijn equation

$$x'(t) = -\frac{x(t-1)}{t}, \qquad t \geq 1,$$

our result yields an explicit asymptotic representation of the solutions as $t \to \infty$. This is a joint work with Gergely Röst (University of Szeged, Hungary).

Reduction of nonlinear integral boundary value problems to a certain model type

Miklós Rontó and Yana Varha
University of Miskolc

We consider the following non-linear integral boundary value problem

$$\frac{dx}{dt} = f\left(t, x\right), t \in [a, b], \int_a^b g(s, x(s))ds = d.$$

Here we suppose that $f : [a, b] \times D \to \mathbb{R}^n$, $g : [a, b] \times D \to \mathbb{R}^n$ are continuous and f is locally Lipschitzian with respect to it's second variable.

Let Ω_a and Ω_b be convex subsets, in which one looks for the initial value $x(a)$ and the value $x(b)$, respectively. The problem is to find a continuosly differentiable solution $x : [a, b] \to D$ with initial value $x(a) = z \in \Omega_a$. We note, that the domain D will be defined by using a certain convex linear combination of subsets Ω_a and Ω_b.

At first we simplify the integral boundary condition and reduce it to some two-point separated linear model-type condition. Namely, we introduce the vectors of parameters

$$z := \mathrm{col}(z_1, z_2, ..., z_n) = x(a), \ \eta := \mathrm{col}(\eta_1, \eta_2, ..., \eta_n) = x(b).$$

Instead of the given integral boundary value problem we will study the following "model-type" two-point BVP with separated parametrized conditions :

$$\frac{dx}{dt} = f\left(t, x\right), t \in [a, b], \ x(a) = z, \ x(b) = \eta.$$

We connect the introduced model type problem with the special parameterized sequence of functions $\{x_m(t, z, \eta)\}_{m=0}^{\infty}$ satisfying the boundary conditions $x(a) = z$, $x(b) = \eta$ for all $z, \eta \in \mathbb{R}^n$.

We prove the uniform convergence of the above sequence of functions to a certain limit function $x_\infty(t, z, \eta) = \lim\limits_{m\to\infty} x_m(t, z, \eta)$ on the interval $t \in [a, b]$. The limit function $x_\infty\left(t, z^*, \eta^*\right)$ will be a solution of the original integral boundary value problem if and only if the pair of parameters (z^*, η^*) satisfies the following system of $2n$ algebraic equations:

$$[\eta - z] - \int_a^b f(s, x_\infty\left(s, z, \eta\right))ds = 0, \ \int_a^b g(s, x_\infty\left(s, z, \eta\right))ds - d = 0.$$

Global dynamics in a commodity market model of Bélair and Mackey

Gergely Röst
Bolyai Institute, University of Szeged

We study the global behavior of the price dynamics in a commodity market governed by a balance between demand and supply. While the dependence of demand on price is considered instantaneous, the supply term contains a delay, leading to a delay differential equation. A discrete model is naturally defined as a limit case of this equation. We provide a thorough study of the discrete case, and use these results to get new sufficient conditions for the global convergence of the solutions to the positive equilibrium in the continuous case. When the equilibrium is unstable, we provide some bounds for the amplitude of the oscillations that are quite sharp when the delay is large.

Joint work with Eduardo Liz (Vigo).

Second order quasilinear functional evolution equations

László Simon
Institute of Mathematics, Eötvös Loránd University

We consider an "abstract" second order quasilinear evolution equation where certain terms contain functional dependence on the unknown function. Existence, uniqueness and qualitative properties of the solutions are shown. The general results are applied to different types of quasilinear partial functional equations and systems.

Oscillations in adaptive network processes

Péter L. Simon
Institute of Mathematics, Eötvös Loránd University

Epidemic propagation is considered on an adaptive network, where edges are activated and deleted randomly depending on the states of the nodes connected by them. The resulting model behaviour is explored via numerical simulation and by using an approximating system of ODEs. Unlike on a static network, the spectrum of behaviour is more complex with the number of infecting nodes exhibiting not only a single steady-state but also bistability and oscillations. The ODE systems shows three bifurcations: transcritical, saddle-node and Andronov–Hopf bifurcation. It is investigated how the bifurcation diagram of the ODE translates to the classification of behaviours of the continuous time Markov chain, which is the exact model of the process, especially, the existence of oscillations is studied.

One-parameter groups of Möbius transformations and delayed ODE

Gábor Stépán
Budapest University of Technology and Economics

Nonlinear effects in cutting processes described by delay-differential equations result in parameter domains (called bistable or unsafe zone) where stable oscillation (called chatter) occurs together with stable trivial solution (called stationary cutting). A measurement method is proposed to identify the 'size' of this bistable zone as function of one of the relevant parameters (the chip thickness). The constant width of this bistable zone corresponds to the classical power law, while a global maximum of the width of this zone refers to the existence of an inflexion in the cutting force characteristics.

One-parameter groups of Möbius transformations and delayed ODE

László Stachó
Bolyai Institute, University of Szeged

We study and formulate some problems concerning the differential equations associated with pointwise continuous one-parameter groups of non-linear holomorphic automorphisms of the unit ball in a Hilbert space.

Detailed balance and microscopic reversibility in deterministic and stochastic chemical kinetics

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The two terms in the title are usually used as synonyms describing a general principle. However, if one tries to give rigorous definitions at least within a class of models (namely the models of reaction kinetics) it turns out that the natural formulations differ if one works with a deterministic or a stochastic model. A series of examples taken from deterministic and stochastic reaction kinetics will show the relations, including examples showing that if the stochastic model of a reaction obeys the principle its deterministic model may not obey it. Our conjecture is just the opposite what might be expected: fulfilment of the principle in the deterministic model implies that it holds in the stochastic case, as well.

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Non-monotone travelling waves in the Mackey–Glass type and the Hutchinson diffusive equations

Sergei Trofimchuk
Universidad de Talca, Chile

We compare the geometric forms of travelling waves in the Mackey–Glass type diffusive equations with the shapes of waves for the delayed KPP-Fisher equation. We show that, similarly to what happens with the original (i.e. non-diffusive) models, the Mackey–Glass type diffusive equations possess a richer variety of non-monotone waves.

Spectrum and amplitude equations for DDEs with large delay

Serhiy Yanchuk
Humboldt University of Berlin, Germany

Firstly, I describe the asymptotic properties of the spectrum of linear DDEs with one large delay. Using these properties, one can classify possible types of destabilization of steady states. In the limit of large delay, this classification is similar to the one for parabolic partial differential equations. Then I present error estimates for amplitude equations, which describe universally the local behavior of scalar delay differential equations close to the destabilization threshold. Finally, some results for two-delay equations will be mentioned.

On the global attractor of delay equations with positive feedback

Gabriella Vas
MTA–SZTE Analysis and Stochastics Research Group, University of Szeged

Consider the scalar delay equation

$$\dot{x}(t) = -\mu x(t) + f(x(t-1)),$$

where $\mu > 0$ and f is a strictly increasing C^1-function with $f(0) = 0$ (positive feedback case).

According to the previous results of Krisztin, Walther and Wu, if the equation admits one unstable equilibrium point between two stable ones in terms of pointwise ordering, then the global attractor – under mild technical conditions – is a spindle-like structure. In this talk we investigate the structure of the global attractor when more equilibria exist. In this case large-amplitude periodic orbits may appear outside the spindles. We give a geometrical description for the unstable set of a specific large-amplitude periodic orbit.

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Complicated histories due to variable delay

Hans-Otto Walther
University of Giessen, Germany

There is a delay functional d which equals 1 near $0 \in C([-2,0],\mathbb{R})$ so that the equation

$$x'(t) = -\alpha\, x(t - d(x_t)),$$

with 2-dimensional unstable space at the origin, has a solution h which is homoclinic to 0, and along its flowline $t \mapsto h_t$ the intersection of stable and unstable manifolds is minimal. We show that close to the homoclinic loop there exists a kind of chaotic motion which involves partially unstable behaviour of solutions, without periodic orbits. The proof does not make use of any relation between growth in the unstable manifold and decay in the stable manifold.

Poster presentations

Hopf bifurcation and period functions for Wright type delay differential equations

István Balázs, Gergely Röst
Bolyai Institute, University of Szeged

In this note, we provide the simplest criterion that determines the direction of the Hopf bifurcations of the delay differential equation $x'(t) = -\mu f(x(t-1))$, as the parameter μ passes through the critical values μ_k. Using this information and the Cooke transformation, we obtain local estimates on the periods of the bifurcating limit cycles along the Hopf branches.

Delay equations explain Quorum sensing of *P. putida*

Maria Vittoria Barbarossa[1] and Christina Kuttler[2]
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[2]Technische Universität München, Munich, Germany

The bacterial strain *Pseudomonas putida IsoF*, isolated from a tomato rhizosphere, possesses a Quorum sensing regulation system, which allows the bacteria to recognize aspects of their

environment or to communicate with each other by the so-called autoinducer molecules. In an experimental study the time series of the autoinducer (AHL) production did not show the expected behavior, as it was observed for other bacterial species by indirect measurements. Our approach supports the hypotheses of the existence of a further enzyme, which degrades the AHLs into an inactive form. As numerical simulations show, the delay model can explain the AHL dynamics observed in the experiments, thus supporting the biological hypotheses. Further we could prove that the system shows a typical bistable behavior, choosing, e.g., bacterial population density or abiotic degradation rate as exemplary bifurcation parameters. With a particular choice of the parameter values in the delay model an oscillatory behavior was found.

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Dynamic control of network-based epidemic models

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We apply nonlinear model predictive control (NMPC) to a pairwise ODE model which we use to model a susceptible-infectious-susceptible (SIS) epidemic on non-trivial contact structures. While classic control of epidemics concentrates on aspects such as vaccination, quarantine and fast diagnosis, our novel setup allows us to deliver control by altering the contact network within the population. Moreover, the ideal outcome of control is to eradicate the disease while keeping the network well connected. We investigate numerically the impact and interaction of system and control parameters on the controllability of the system.

Interval maps with hysteresis

Rudolf Csikja
Budapest University of Technology and Economics

We investigate the statistical and topoligical properties of expanding interval maps with hysteresis. Particularly, we study the beta-hysteresis transformation, which is the hysteretic version of the beta-transformation, and it can be interpreted as a Poincaré map of a hybrid dynamical system with hysteresis.

On the stabilization of the inverted pendulum by a geometric method

László Csizmadia
Bolyai Institute, University of Szeged

Sufficient conditions are given for the stability of the upper equilibrium of the inverted pendulum when the suspension point is vibrating with high frequency. M. Levi and W. Weckesser gave a simple geometrical explanation for the stability effect provided that the frequency is so high that the gravity can be neglected. They also obtained a lower estimate for the stabilizing frequency. This method and this estimate are improved and extended to the arbitrary inverted pendulum under the effect of gravity not assuming even simmetricity between the upward and downward phases in the vibration of the suspension point.

Risk of infectious disease outbreaks by imported cases with application to the European Football Championship 2012

Attila Dénes[1], Péter Kevei[2], Hiroshi Nishiura[3] and Gergely Röst[1]
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[3]University of Tokyo, Tokyo, Japan

The European Centre for Disease Prevention and Control called the attention in March 2012 to the risk of measles in Ukraine among visitors to the 2012 UEFA European Football Championship. Large populations of supporters travelled to various locations in Poland and Ukraine, depending on the schedule of Euro 2012 and the outcome of the games, possibly carrying the disease from one location to another. We propose a novel two-phase multitype branching process model with immigration to describe the risk of a major epidemic in connection with large-scale sports-related mass gathering events. By analytic means, we calculate the expected number and the variance of imported cases and the probability of a major epidemic caused by the imported cases in their home country. Applying our model to the case study of Euro 2012, we demonstrate that the results of the football games can be highly influential to the risk of measles outbreaks in the home countries of supporters. To prevent imported epidemics, it should be emphasized that vaccinating travelers would most efficiently reduce the risk of epidemic, while requiring the minimum doses of vaccines as compared to other vaccination strategies. Our theoretical framework can be applied to other future sport tournaments too.

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Global stability of some second order difference equations

Ábel Garab
Bolyai Institute, University of Szeged

Consider the second order difference equation

$$x_{k+1} = x_k e^{\alpha - x_{k-d}},$$

where α is a positive parameter and d is a nonnegative integer. The case $d = 0$ was introduced by W. E. Ricker in 1954. For the delayed version $d \geq 1$ of the equation S. Levin and R. May conjectured in 1976 that local stability of the nontrivial equilibrium implies its global stability. Based on rigorous, computer-aided calculations and analytical tools, we prove the conjecture for $d = 1$. We also apply our method to give necessary and sufficient conditions for the global stability of the trivial equilibrium of the difference equation $x_{k+1} = mx_k + \alpha \tanh x_{k-1}$, where m and α are real parameters. Joint work with Ferenc Bartha and Tibor Krisztin.

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Traveling waves in the nonlocal KPP–Fisher equation

Karel Hasík
Silesian University, Opava, Czech Republic

We consider a nonlocal KPP–Fisher equation. The non-locality is expressed in terms of convolution with a kernel $K(s) \geq 0, s \in \mathbb{R}$, $\int K(s)ds = 1$. We show that restrictions $K(s), s \geq 0$, and $K(s), s \leq 0$, play quite different roles as for the existence and uniqueness of traveling fronts to the KPP–Fisher equation. In particular, the prevalence of the left interaction assures the uniqueness of fronts while the increase of the right interaction can induce the co-existence of monotone and oscillating fronts in our KPP–Fisher equation. We also present a short proof of the existence of traveling waves without assuming various technical restrictions usually imposed on K.

Cross-diffusion modeling in macroeconomics

László Balázsi, Krisztina Kiss
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Budapest University of Technology and Economics

We deal with the stability properties of a closed market, where capital and labour force are acting like a predator–prey system in population–dynamics. The spatial movement of the capital and labour force are taken into account by cross-diffusion effect. This is modeled by a reaction-diffusion system inside a country, while it is modeled by patches between two countries. We prove that if these effects are superimposed then some additional conditions have

to be satisfied in order to guarantee the stability of the spatially constant solution, compared to those cases when these effects were taken into account separately. Our four-dimensional model highlights that a hectic movement of the capital toward labour force can cause a Turing instability.

Epidemic spread and variation of peak times in connected regions due to travel related infections

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National boundaries never hindered infectious diseases to reach distant territories, however the speed at which an infectious agent can spread around the world via the global airline transportation network has significantly increased during the last decades. We introduce an SEAIR-based, antigravity-type model to investigate the spread of an infectious disease in two regions, which are connected by transportation. As a submodel, an age-structured system is constructed to incorporate the possibility of disease transmission during travel, where age is the time elapsed since the start of the travel. The model is equivalent to a large system of differential equations with dynamically defined delayed feedback. After describing fundamental, but biologically relevant properties of the system, we detail the calculation of the basic reproduction number and obtain disease transmission dynamics results in terms of R_0. We parameterize our model for influenza and use real demographic and air travel data for the numerical simulations. To understand the role of the different characteristics of the regions played in the propagation of the disease, three distinct origin-destination pairs are considered. The model is also fitted to the first wave of the A(H1N1) 2009 pandemic influenza in Mexico and Canada. Our results highlight the importance of including travel time and disease dynamics during travel in the model: the invasion of disease free regions is highly expedited by the elevated transmission potential during transportation.

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Error analysis of waveform relaxation for partial differential equations

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The method of waveform relaxation is examined for partial differential equations in a general framework, namely without spatial or temporal discretization. The interaction between

the iteration error and the numerical error is investigated. In practice the iterative subproblems are solved numerically, an explicit upper bound for the cumulative numerical error is given. A numerical example is presented to illustrate the behavior of errors arising from different sources.

Internal stability of SSP and extrapolation methods

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Runge–Kutta methods are widely used for solving initial value problems for ODEs. In practical computations the stage equations within a single iteration step are not always satisfied exactly due to roundoff errors or algebraic solver errors. Amplification of such errors is an increasingly important issue because there now exist several classes of practical Runge–Kutta methods that use many stages, including some strong stability preserving (SSP) methods, or extrapolation methods.

We provide bounds on the internal error amplification constants for optimal second and third order explicit SSP methods with s stages, and for extrapolation methods of order p based on the explicit Euler method or the explicit midpoint method. In our investigations s and p can be arbitrarily large.

The above results are based on joint work with David Ketcheson (KAUST) and Matteo Parsani (NASA Langley Research Center).

Modeling of *P. vivax* malaria with bimodal incubation time

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Malaria parasites are transmitted between mosquitoes and humans. If an infectious mosquito bites a host, symptoms occur after a certain incubation period. The incubation period can vary depending on the species of parasite or the regions. In particular, incubation period of *Plasmodium vivax* – the malaria inducing parasite species most prevalent in temperate zones in Korea – shows bimodal distribution, with short term and long-term incubation periods. In this poster presentation, we compare transmission models for *P. vivax* malaria having different expression for the incubation period.

Global dynamics of two-compartment models for cell production systems with regulatory mechanisms

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We present a global stability analysis of two-compartment models of a hierarchical cell production system with a nonlinear regulatory feedback loop. The models describe cell differentiation processes with the stem cell division rate or the self-renewal fraction regulated by the number of mature cells. Using global stability analysis, we compare different regulatory mechanisms. For both models, we show that there exists a unique positive equilibrium that is globally asymptotically stable if and only if the respective reproduction numbers exceed one.

Existence of positive solutions of linear delay difference equations with continuous time

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In this paper linear delay difference equations with continuous time of the form

$$x(t) - x(t-1) + \sum_{i=1}^{m} P_i(t)x(t-k_i) = 0, \quad t \geq t_0$$

is considered, where $P_i \in C[[t_0, \infty), \mathbf{R}]$ and $k_i \in \mathbf{N}$ such that $k_i > 1$, $i = 1, 2, \ldots, m$. We introduce the generalized characteristic equation and their importance at oscillation of all solutions of considered difference equations. Some results for the existence of positive solutions of considered difference equations are presented as the application of the generalized characteristic equation.

Endemic bubbles generated by delayed behavioral response in epidemic models

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Many attempts have been made in the past to capture the phenomenon that people modify their behavior during an epidemic outbreak. This reaction can be triggered by directly experiencing the rising number of infections, media coverage, or intervention policies. In this talk we

show that a delayed activation of such a response can lead to some surprising dynamics. For an SIS type process, if the delayed response occurs with a jump in the contact rate when the density of infection reaches some threshold, we show that for some interval of reproduction numbers, the system is oscillatory. The oscillation frequency is a discrete Lyapunov functional and there exists a unique slowly oscillatory periodic solution with strong attractivity properties. We also construct rapidly oscillatory periodic solutions of any frequency. In the case of continuously decreasing transmission rate, if the response is not too strong, the system preserves global stability. However, for sharp delayed response, we can observe stability switches as the basic reproduction number is increasing. First, the stability is passed from the disease free equilibrium to an endemic equilibrium via transcritical bifurcation as usual, but a further increase of the reproduction number causes oscillations, which later disappear for higher values of the reproduction number, forming an interesting structure in the bifurcation diagram what we call *endemic bubble*.

Can Brazilian waxing kill the pubic louse?

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The pubic louse is an obligate ectoparasite of humans which lives mostly in pubic hair. Lice infect a new host only by close contact between individuals, usually through sexual activity. According to recent media reports, the crab louse (*Pthirus pubis*) has almost been driven to extinction in several countries. This was attributed to the gaining popularity of pubic hair removal among females, which also provides protection againt lice infestation. To capture this interesting phenomenon, we introduce a mathematical model, that is SIS type for males and $SIRS$ type for females, where the R compartment contains females having no (or only a little) pubic hair. We identify the basic reproduction number R_0 which turns out to be a threshold parameter and give an analysis of the global dynamics via Lyapunov functions. Finally, by parameter estimation and data analysis we try to answer the question whether the cultural phenomenon of pubic hair removal can indeed eradicate *Pthirus pubis*.

Backward bifurcation for pulse vaccination

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We investigate the types of appearing bifurcations in SIVS model with pulse vaccination strategy. First we compute the disease-free periodic solution and prove its global asymptotic stability in the disease-free subspace. We identify the corresponding control reproduction number R_c and prove that the disease-free periodic solution is locally asymptotically stable if $R_c < 1$. Furthermore, under some additional conditions it is globally asymptotically stable, too. For $R_c > 1$ we prove the uniform persistence of the disease. Our main result is that non-trivial endemic periodic solutions are bifurcating from the disease free periodic solution as R_c

passes the threshold value one. A complete bifurcation analysis is provided for the associated nonlinear fixed point equation. We show that backward bifurcation of periodic orbits is possible, and give explicit conditions to determine whether the bifurcation is backward or forward. The main mathematical tools are comparison principles and Lyapunov–Schmidt reduction.