Applied Mathematics MSc Final Oral Exam

The Final Exam consists of two parts: the MSc thesis defense, and the MSc final oral exam. For the final oral exam there are 15 topics in each specialization. The students can choose 5 topics (out of the 15) that they want to avoid, beforehand, prior to the exam. At the beginning of the exam they draw one topic from the remaining 10. The oral exam will be in this topic. Before the oral exam there is a two hours long preparation in the library where it is allowed to use the printed literature.

Topics for the final oral exam

Applied Analysis Specialization

1. Algorithms and their complexity
Dynamic programming, flow algorithms, amortized analysis of algorithms, data structures, heaps, LZW data compression algorithm, Turing machines, nondeterministic computation, complexity classes and their relations, reductions and complete problems, NP-completeness, examples, randomized algorithms.

2. Differential equations
Existence, uniqueness, smoothness of solutions of initial value problems. Linear autonomous systems. Nonlinear systems. Stability: linearization, Lyapunov theorems. Linear partial differential equations: transport equation, heat equation, wave equation.

3. Numerical solutions of differential equations
Ordinary differential equations: Euler and Runge-Kutta methods, finite element method. Numerical methods for the diffusion and heat equation, explicit, implicit methods, finite element method, Dirichlet and Neumann boundary conditions; the wave equation by finite difference method, the CFL condition, finite element solution.

4. Discrete mathematics
Cayley's theorem (enumeration of trees), k-connected and k-edge connected graph, theorems of Menger, edge colorings and Vizing's theorem, matchings in graphs, planarity.

5. Fourier series, orthogonal polynomials, series expansions
Trigonometric polynomials, orthogonal polynomial systems, convergence of Fourier series, Fourier and Laplace transform.

6. Functional analysis
Banach and Hilbert spaces. Bounded linear operators and functionals. The fundamental theorems of linear functional analysis (the open mapping theorem, the closed graph theorem, the uniform boundedness principle). Spectrum of bounded operators, the spectral theorem of self-adjoint compact operators.

7. Time series
Stationary sequences, autocorrelation and partial autocorrelation functions, spectral theory, ARMA processes and statistics, Yule-Walker equations.

8. Types of convergence, convergence in distribution
Characteristic functions, types of convergence, Portmanteau theorem, central limit theorem, multidimensional normal distribution, renewal theory, Poisson process.

9. Optimization methods
Optimization problems, examples and special classes, Lagrangian duality, weak and strong duality theorems, Karush-Kuhn-Tucker theorem, combinatorial applications of LP and SDP problems, branch and bound methods.

10. Random variables, expectation, laws of large numbers
Random variables and vectors, distribution function, expectation, moments, independence, Borel-Cantelli lemmas, Kolmogorov 0-1 theorem, laws of large numbers.

11. Functions of several variables and vector valued functions
Multiple integral, line integral, surface integral, Green theorem, Gauss theorem, Stokes theorem, applications of integrals in physics and engineering.

12. Dynamical systems
Stability. Invariant manifolds. Elementary bifurcations. Chaos. Symbolic dynamics. Limit sets. The Poincaré-Bendixson theorem. Poincaré map. Stability of periodic orbits, orbital stability. Structural stability, genericity. Hartman-Grobman theorem.

13. Numerical mathematics
Eigenvalues of matrices by QR iteration and inverse power method. Multidimensional Newton iteration. Optimization using STP and conjugate gradient method. Moore-Penrose inverse, spline regression. Interval arithmetic.

14. Partial differential equations
Representation formulas for the Laplace, heat and wave equations. Distributions, Sobolev spaces. Variational formulation for elliptic PDEs. Weak solutions. The Lax-Milgram theorem.

15. Topology and manifolds
Topological spaces, convergence, metric spaces, compactness, completeness. Differentiable manifolds, Riemann-metric and covariant differentiation, curvatures. Differential forms and the Gauss-Bonnet Theorem.

Industrial Mathematics Specialization

1. Algorithms and their complexity
Dynamic programming, flow algorithms, amortized analysis of algorithms, data structures, heaps, LZW data compression algorithm, Turing machines, nondeterministic computation, complexity classes and their relations, reductions and complete problems, NP-completeness, examples, randomized algorithms.

2. Differential equations
Existence, uniqueness, smoothness of solutions of initial value problems. Linear autonomous systems. Nonlinear systems. Stability: linearization, Lyapunov theorems. Linear partial differential equations: transport equation, heat equation, wave equation.

3. Numerical solutions of differential equations
Ordinary differential equations: Euler and Runge-Kutta methods, finite element method. Numerical methods for the diffusion and heat equation, explicit, implicit methods, finite element method, Dirichlet and Neumann boundary conditions; the wave equation by finite difference method, the CFL condition, finite element solution.

4. Discrete mathematics
Cayley's theorem (enumeration of trees), k-connected and k-edge connected graph, theorems of Menger, edge colorings and Vizing's theorem, matchings in graphs, planarity.

5. Fourier series, orthogonal polynomials, series expansions
Trigonometric polynomials, orthogonal polynomial systems, convergence of Fourier series, Fourier and Laplace transform.

6. Functional analysis
Banach and Hilbert spaces. Bounded linear operators and functionals. The fundamental theorems of linear functional analysis (the open mapping theorem, the closed graph theorem, the uniform boundedness principle). Spectrum of bounded operators, the spectral theorem of self-adjoint compact operators.

7. Time series
Stationary sequences, autocorrelation and partial autocorrelation functions, spectral theory, ARMA processes and statistics, Yule-Walker equations.

8. Types of convergence, convergence in distribution
Characteristic functions, types of convergence, Portmanteau theorem, central limit theorem, multidimensional normal distribution, renewal theory, Poisson process.

9. Optimization methods
Optimization problems, examples and special classes, Lagrangian duality, weak and strong duality theorems, Karush-Kuhn-Tucker theorem, combinatorial applications of LP and SDP problems, branch and bound methods.

10. Random variables, expectation, laws of large numbers
Random variables and vectors, distribution function, expectation, moments, independence, Borel-Cantelli lemmas, Kolmogorov 0-1 theorem, laws of large numbers.

11. Functions of several variables and vector valued functions
Multiple integral, line integral, surface integral, Green theorem, Gauss theorem, Stokes theorem, applications of integrals in physics and engineering.

13. Numerical mathematics
Eigenvalues of matrices by QR iteration and inverse power method. Multidimensional Newton iteration. Optimization using STP and conjugate gradient method. Moore-Penrose inverse, spline regression. Interval arithmetic.

13. Theoretical mechanics
Newton’s laws of motion. Vibrations. Motion in a gravitational field. The two-body problem. Kepler’s laws. Motions on the rotating Earth. The n-body problem, the 10 first integrals. The principle of Galilean relativity. Constraints. Lagrange equations. Variational principles. Hamilton equations. Conservation laws.

14. Control theory
Basics of feedback control and applications, examples for input/output response. System modeling and state space representations (time-region), transfer functions (s-region), frequency response (frequency range). Basic control functions, PID (proportional integral-derivative) control, stability of systems.

15. Coding theory
Coding, error detection and error correcting codes. Public-key cryptography.

Financial Mathematics Specialization

1. Algorithms and their complexity
Dynamic programming, flow algorithms, amortized analysis of algorithms, data structures, heaps, LZW data compression algorithm, Turing machines, nondeterministic computation, complexity classes and their relations, reductions and complete problems, NP-completeness, examples, randomized algorithms.

2. Differential equations
Existence, uniqueness and smoothness of solutions of initial value problems. Linear autonomous systems. Nonlinear systems. Stability: linearization, Lyapunov theorems. Linear partial differential equations: transport equation, heat equation, wave equation.

3. Numerical solutions of differential equations
Ordinary differential equations: Euler and Runge-Kutta methods, finite element method. Numerical methods for the diffusion and heat equation, explicit, implicit methods, finite element method, Dirichlet and Neumann boundary conditions; the wave equation by finite difference method, the CFL condition, finite element solution.

4. Discrete mathematics
Cayley's theorem (enumeration of trees), k-connected and k-edge connected graph, theorems of Menger, edge colorings and Vizing's theorem, matchings in graphs, planarity.

5. Fourier series, orthogonal polynomials, series expansions
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6. Functional analysis
Banach and Hilbert spaces. Bounded linear operators and functionals. The fundamental theorems of linear functional analysis (the open mapping theorem, the closed graph theorem, the uniform boundedness principle). Spectrum of bounded operators, the spectral theorem of self-adjoint compact operators.

7. Time series
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8. Types of convergence, convergence in distribution
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9. Optimization methods
Optimization problems, examples and special classes, Lagrangian duality, weak and strong duality theorems, Karush-Kuhn-Tucker theorem, combinatorial applications of LP and SDP problems, branch and bound methods.

10. Random variables, expectation, laws of large numbers
Random variables and vectors, distribution function, expectation, moments, independence, Borel-Cantelli lemmas, Kolmogorov 0-1 theorem, laws of large numbers.

11. Martingales
Conditional expectation, discrete and continuous time martingales, Doob-Meyer decomposition, Doob's maximal inequality, martingale convergence theorem.

12. Wiener process, Markov processes
Wiener process and properties, Markov processes, Chapman-Kolmogorov equations.

13. Stochastic calculus and applications
Stochastic integral, Itô formula, stochastic differential equations, Girsanov's theorem, continuous time markets, Black-Scholes model.

14. Mathematical finance and risk theory
Discrete time markets, portfolio and hedge, arbitrage and completeness, CRR formula, classical risk process, Cramér-Lundberg theorem, Lundberg exponent.

15. Statistics
Sufficiency and completeness, Fisher information, Cramér-Rao inequality, Rao-Blackwell-Kolmogorov theorem, hypothesis testing, Neyman-Pearson lemma, linear regression, analysis of variance.