On a class of neutral equations with state-dependent delay in population dynamics

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Abstract

In the first part of this talk we consider a class of neutral equations with state-dependent delay of the form

$$\dot{x}(t) = \frac{\alpha\left(x(t), x(t-\tau(x(t))), \dot{x}(t-\tau(x(t)))\right) - \gamma(x(t))}{1 + \dot{\tau}(x(t))\alpha\left(x(t), x(t-\tau(x(t))), \dot{x}(t-\tau(x(t)))\right)}, \tag{1}$$

where α, γ are continuously differentiable functions and τ is a non-negative, bounded C^2-function.
Equations of the class (1) satisfy the assumptions of the framework in (Walther, 2011) up to one hypothesis, which we replace by an equivalent condition. As a result, it is possible to formulate a principle of linearized stability for (1).

In the second part of the talk, we apply the results on (1) to the following problem:

$$\dot{x}(t) = \frac{\beta_{t,\tau} - \tilde{\mu}_1(x(t))}{1 + \dot{\tau}(x(t))\beta_{t,\tau}},$$

where

$$\beta_{t,\tau} := \left[\tilde{b}_1(x(t-\tau)) + b_2(x(t-\tau))\frac{\dot{x}(t-\tau) + \tilde{\mu}_1(x(t-\tau))}{1 - \dot{\tau}(x(t-\tau))\dot{x}(t-\tau)}\right] e^{-\mu_0 \tau(x(t))}.$$

This equation expresses the dynamics of a population of mature individuals and has been derived in (Barbarossa, 2012) from an age-structured Gurtin-MacCamy model for population dynamics (Gurtin, 1974).

References

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