

*XXIX-th European Meeting of Statisticians, 2013, Budapest*

**In Memoriam SÁNDOR CSÖRGŐ: an Appreciative Glimpse  
of his Manifold Contributions to Stochastics, a Tribute to  
my brother Sándor**

MIKLÓS CSÖRGŐ \*,†

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<b>Főmenü</b>
<b>Kezdőlap</b>
<b>Településünkéről</b>
Egerfarmos története
Földrajzi adatok
Kulturális, építészeti emlékek
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<b>Elérhetőségek</b>

**Kezdőlap** » **Településünkéről** » **Általános Iskola**

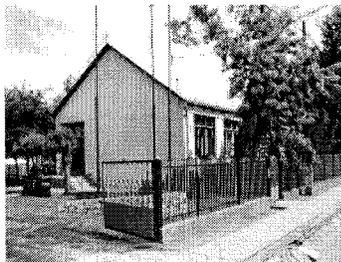
### Általános Iskola



#### Általános Iskola

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Egerfarmoson a legrégebbi, megőrzött iskolai dokumentumok (anyakönyvek, bizonyítványok) a múlt század elejéről származnak. Ezen okiratok tanúsága szerint az 1908-ban született gyermekek már biztosan Egerfarmoson jártak iskolába. Sok idős ember még ma is emlékszik a régi iskolaépületekre, igaz ezek ma már más funkciót töltenek be. (Idősek Klubja, Orvosi Rendelő)



Az az épület, amelyben napjainkban tanulnak a gyerekek, az 1970-es évek elején létesült, két tanteremmel, folyosóval, tanári szobával és sokáig raktárnak használt helyiséggel. Többre sajnos nem volt szükség, mert a '70-es évek közepén olyannyira megfogyatkozott a lakosság és ezáltal a gyermekek száma is, hogy a felső tagozatosok 1975-től Mezőszemerére jártak. Az intézmény pedig a mezőszemerei Általános Iskola tagiskolájává vált, s ez így volt 1990-ig.

A rendszerváltozás új fejezetet nyitott az intézmény életében, ismét önállóvá válhatott. Fontos feladatának tartotta az akkori kormányzat, valamint a helyi vezetőség is, hogy a kisiskolák fejlődjenek, tartsanak lépést a korrallal. Így több, nagyobb léptékű beruházás történt, helyben is. Vizesblokk került kialakításra, korszerűsítették az épület fűtését is. Modern-jelen viszonyoknak is megfelelő-tanulói bútorzat került a tantermekbe.

Napjainkban már számítógépes ismeretekkel is bővül diákjaink tudása, a világháló számunkra is elérhető. Iskolai könyvtárunknak köszönhetően minden gyerek olyan ismeretanyaghoz jut, ami számára megfelelő, érdeklődési köréhez tartozik. Az első idegen nyelv oktatására már több éve lehetősége van az intézménynek. Önkormányzatunk segítségével minden évben úszásoktatást is szervezünk tanulóink részére, hiszen ez igen fontos és természetesen egészségmegőrző szerepe is hangsúlyos eme sportnak.

Nincs olyan tanév, amikor a kikapcsolódás mellett, hasznos tudnivalókkal, látnivalókkal gazdagodjanak a tanulók.

Kirándulásaink, nyári táboraink mind-mind azt a célt szolgálják, hogy e kis településen élő gyermekek is bejárják hazánk ismert és kevésbé ismert helyeit, s megismerkedjenek a főváros látnivalóival is.

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Bízunk abban, hogy iskolánkat még évekig megőrizhessük, hiszen a végzős negyedikesek szép eredménnyel bizonyítják tudásukat a felső tagozatos osztályokban és a középiskolában is. Hisszük, hogy e tudás alapját mi raktuk le itt Egerfarmoson.

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Utolsó frissítés ( 2007. július 01. vasárnap 11:29 )

Ma 2013. június 19. szerda,  
Gyárfás napja van.

#### Önkormányzat

#### Álláshirdetések

#### Közadatkereső

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VIII. Egerfarmosi  
hagyományörző napok









*„A tudomány nem ismer széles országutakat, s csak azok remélhetik, hogy napsütötte ormait elérik, akik nem riadnak vissza attól, hogy meredek ösvényeinek megmászása fáradságos.”*

(Marx: Tőke)

SZERETETTEL MEGHÍVOM

ÖNT ÉS B. CSALÁDJÁT

1965. MÁJUS HÓ 8 N 10 ÓRAKOR

TARTANDÓ

*BALLAGÁSI ÜNNEPSÉGÜNKRE*



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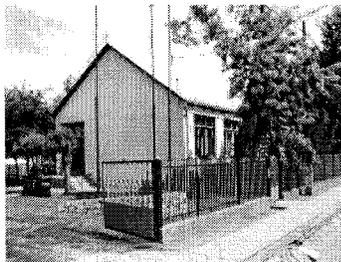
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VIII. Egerfarmosi  
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# Randomly indexed invariance theorems

Csörgő M, Csörgő S

AN INVARIANCE PRINCIPLE FOR THE EMPIRICAL PROCESS WITH RANDOM SAMPLE SIZE. *BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY* 76: pp. 706–710. (1970)

ON WEAK CONVERGENCE OF RANDOMLY SELECTED PARTIAL SUMS. *ACTA SCIENTIARUM MATHEMATICARUM - SZEGED* 34. pp. 53–60 (1973)

Csörgő S

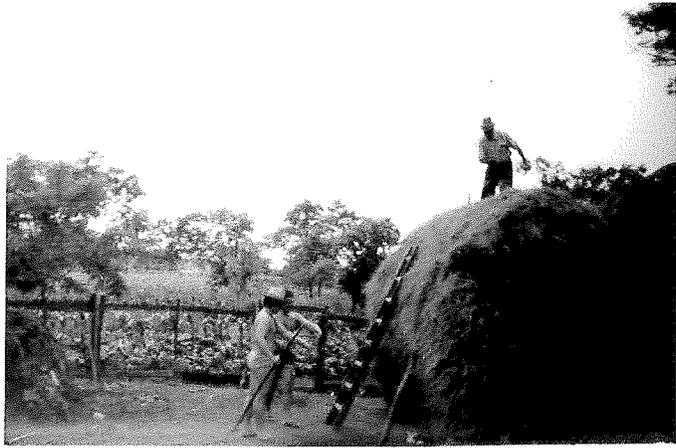
ON WEAK CONVERGENCE OF EMPIRICAL PROCESS WITH RANDOM SAMPLE SIZE *ACTA SCIENTIARUM MATHEMATICARUM - SZEGED* 36: pp. 26–25 Correction: 374–375. (1974)

ON LIMIT DISTRIBUTIONS OF SEQUENCES OF RANDOM VARIABLES WITH RANDOM INDICES *ACTA MATHEMATICA HUNGARICA* 25 pp. 226–232. (1974)

Csörgő M, Csörgő S, Fishler R, Révész P

VÉLETLEN-INDEXES HATÁRELOSZLÁSTÉTELEK ERŐS INVARIANCIA-TÉTELEK SEGÍTSÉGÉVEL *MATEMATIKAI LAPOK* 26: pp. 39–66. (1975)





**MR0494375 (58#13249) 60B10 (60F05)**

**Csörgő, Miklós; Csörgő, Sándor; Fischler, Roger; Révész, Pál Random limit theorems by means of strong invariance principles. (Hungarian. English summary)**

*Mat. Lapok* **26** (1975), no. 1-2, 39-66 (1977).

... The limit theorems proved in the paper concern either empirical and quantile processes when the sample size is random, or partial sums of a random number of independent random variables.

Section 1 of the paper is introductory and also contains some historical remarks. Section 2 deals with weak and strong laws of large numbers. Section 3 gives weak and strong invariance theorems for randomly selected partial sums. In Section 4 invariance theorems are proved for univariate and multivariate empirical and quantile processes with random sample size. The corresponding limit processes in Section 3 and Section 4 are Wiener process and Brownian bridge and Kiefer process, respectively. Finally, in Section 5 of limit of  $\nu_n/f(n)$  is investigated, where  $\nu_n$  is either the random number of summands or the random sample size. It is shown, among other results, that if  $\nu_n$  is the index of the first maximum in the sequence of partial sums of i.i.d. random variables, then there does not exist a sequence  $f(n)$  such that  $\nu_n/f(n) \rightarrow \nu$  in probability for some positive random variable  $\nu$ . This clear and well-written paper, containing many interesting and important results and equipped with a bibliography of 69 items, is strongly recommended for translation.

Reviewed by *Endre Csáki*

Chapter 7 in

Csörgő, M., Révész, P. (1981). *Strong approximations in probability and statistics*, Academic Press, New York, NY,  
is based on the above paper.





Csörgő S

Meeting a free man: A snapshot of A.V. Skorokhod from 1972.

In: Portenko M. Syta H (ed.)

Appendix in Anatolii Volodymyrovych Skorokhod: Biobibliografiya

Kiev: Mathematical Institute, Ukrainian National Academy of Sciences, 2005, pp. 111-125.

Also published in

*Technical Report Series of the Laboratory for Research in Statistics and Probability, No. 418 -*  
May 2005, Carleton University-University of Ottawa.



## Rényi-mixing of occupation times\*

Sándor Csörgő

Dept. of Statistics, University of Michigan, Ann Arbor MI 48109-1027, U.S.A.  
Bolyai Institute, University of Szeged, Aradi vértanúk tere 1, 6720 Szeged,  
Hungary

,Hullatja levelét az idő vén fája,  
Terítve hatalmas rétegben alája;  
Én az avart jártam; tűnődve megálltam:  
Egy régi levélen ezt írva találtam.'

Arany

It is shown that the occupation times of a bounded interval by sums of independent and identically distributed random variables are Rényi-mixing under the classical necessary and sufficient condition of Darling and Kac for the original limit theorem. Some consequences are derived for the occupation times of a random walk in a random number of steps, along with an extension of the Darling-Kac theorem for Révész-dependent sequences of random variables.

## 1. INTRODUCTION

Following Rényi [17] and Rényi and Révész [19], we say that a sequence  $\{\xi_n\}_{n=1}^{\infty}$  of random variables, given on a probability space  $(\Omega, \mathcal{E}, \mathbb{P})$ , is mixing with the limiting distribution function  $H(\cdot)$  if  $\mathbb{P}\{\{\xi_n \leq y\} \cap E\} \rightarrow H(y)\mathbb{P}\{E\}$  at every continuity point  $y$  of  $H$  on the real line  $\mathbb{R}$ , for each event  $E \in \mathcal{E}$ , as  $n \rightarrow \infty$ .

## Estimating the tail index

Sándor Csörgő<sup>a,b</sup> and László Viharos<sup>b</sup>

<sup>a</sup>Department of Statistics, Univ. of Michigan, Ann Arbor MI 48109-1027, U.S.A.

<sup>b</sup>Bolyai Institute, University of Szeged, Aradi vértanúk tere 1, 6720 Szeged, Hungary

*Dedicated to Miklós Csörgő*

Two very general classes of estimators have been proposed for the tail index of a distribution with a regularly varying upper tail. One, by S. Csörgő, Deheuvels and Mason (1985), is the class of kernel estimators, the other, by Viharos (1997), is the class of universally asymptotically normal weighted doubly logarithmic least-squares estimators. ... While presenting the main ideas, we also review a substantial part of the literature.

INDEED, there are 138 papers listed in the REFERENCES.

## 1. INTRODUCTION

For a constant  $\alpha > 0$ , let  $\mathcal{R}_n$  be the class of all probability distribution functions  $F(x) = P\{X \leq x\}$  for  $x$  on the real line  $\mathbb{R}$  such that  $1 - F$  is regularly varying at infinity with index  $-1/\alpha$ , that is,

$$1 - F(x) = \frac{\ell(x)}{x^{1/\alpha}}, \quad 1 \leq x < \infty, \quad (1a)$$

where  $\ell$  is a positive function on  $[1, \infty)$ , slowly varying at infinity ...

## RATES OF CONVERGENCE FOR $\omega_n^2$

Let  $V_{n,1}(x)$ ,  $x \in \mathbb{R}$ , be the distribution function of the classical **Cramér-von Mises statistic**  $\omega_n^2 := \int_0^1 \alpha_n^2(y)dy$ , based on independent uniformly distributed random variables on the unit interval  $[0, 1]$ , let  $V_1(x)$  be the limiting distribution function, as  $n \rightarrow \infty$ , and put

$$\Delta_{n,1} = \sup_{0 < x < \infty} |V_{n,1}(x) - V_1(x)|.$$

Csörgő S

### ASYMPTOTIC EXPANSION FOR THE LAPLACE TRANSFORM OF THE VON MISES OMEGA-2 CRITERION

*Teoriya Veroyatnostei i ee Primeneniya* 20: pp. 158–161 (1975)

### ON AN ASYMPTOTIC EXPANSION FOR THE VON MISES OMEGA-2 STATISTIC

*Acta Scientiarum Mathematicarum-Szeged* 38: pp. 45-67 (1976)

In this paper it is shown via KMT (1975) that  $\Delta_{n,1} = O(n^{-1/2} \log n)$  and, on the basis of his complete asymptotic expansion for the Laplace transform of the Omega-2 statistic, **Sándor conjectures**  $\Delta_{n,1} = O(1/n)$ .

Csörgő S, Stachó L.

### A STEP TOWARD AN ASYMPTOTIC EXPANSION FOR THE CRAMÉR VON MISES STATISTIC

*Analytic function methods in probability theory*; (Proc. Colloq. Methods of Complex Anal. in the theory of Probab. and Statist., Kossuth L. Univ. Debrecen, 1977) pp. 53-65, Colloq. math. Soc. János Bolyai; 21, North-Holland, Amsterdam. NY, 1979

Conjecture further studied by giving a recursion formula for the distribution function  $V_{n,1}$  of the Omega-2 statistic.

MR642735 (83d:62078) 62H10 (60H15)

Cotterill, Derek S.; Csörgő Miklós

**On the limiting distribution of and critical values for the multivariate Cramér-von Mises statistic.**

*Ann. Statist.* **10** (1982), no. 1 233-244.

Let  $V_{n,d}(x)$ ,  $x \in \mathbb{R}$ , be the distribution function of the classical Cramér-von Mises statistic based on  $n$  independent  $d$ -dimensional random vectors distributed uniformly on the unit cube of  $\mathbb{R}^d$ , and let  $V_d(x)$  be the limiting distribution function of  $V_{n,d}(x)$  as  $n \rightarrow \infty$ . The authors deduce from the basic results of F. Götze *Z. Wahrsch. Verw. Gebiete* **50** (1979), no. 3, 333-355; MR0554550 (81c:60025)] that  $\sup\{|V_{n,d}(x) - V_d(x)| : x \in \mathbb{R}\} = O(n^{-1})$  for any  $d \geq 1$ . They give recursive formulae for all the moments and cumulants corresponding to the limiting  $V_d$  and use the Cornish-Fisher asymptotic expansion, based on the first six cumulants, to compile tables of the critical values of  $V_d$  corresponding to the usual testing levels. These tables run from  $d = 2$  to 50, and such tables were previously known only for  $d = 1, 2, 3$ . An interesting finding is that  $K_{k,d} = O(e^{-d})$  as  $d \rightarrow \infty$  for the  $k$ th cumulant  $K_{k,d}$ . Consequently, the tables presented become more and more precise as the dimension grows.

Reviewed by *Sándor Csörgő*

In 1979 Sándor was a **Visiting Research Fellow** at Carleton University. The above paper was under revision for publication at that time. Our revision benefited from Sándor's vast knowledge of the area in hand.

MR542130 (84c:62038) 62E20 (60F05) (62G30)

**Burke, M.D.; Csörgő M.; Csörgő. S.; Révész, P.**

**Approximations of the empirical process when parameters are estimated.** *Ann. Probab.* **7** (1982), no. 5 790-810.

The authors use the strong approximation methodology developed by the Hungarian school (see, e.g., J. Komlós et al. [*Z. Wahrsch. Verw. Gebiete* **32** (1975), 111-131; MR0375412 (51#11605b)]) to derive almost sure and in-probability approximation results for the empirical process when parameters are estimated from the data. This approach has been demonstrated in two earlier papers [M. Csörgő et al., *Transactions of the Seventh Prague Conference on Information Theory, Statistical Decision Functions, Random Processes and of the Eighth European Meeting of Statisticians* (Prague, 1974), Vol. B, 87-97, Academia, Prague, 1978; MR0519466 (80c:60033); Burke and M. Csörgő, *Empirical distributions and processes* (Oberwolfach, 1976), 1-16, Lecture Notes in Math., 566, Springer Berlin, 1976; MR0443171 (56 #1542)].

From the introduction: “In this exposition we follow the same road (correcting also previous oversights as we proceed), but we also weaken substantially the regularity conditions under which these representations will hold. As to the type of estimation of the parameters we follow J. Durbin [Ann. Statist. **1** (1973), 179-190; MR0359131 (50 #11586)], and in addition to his weak convergence, we obtain explicit representations of the limiting Gaussian process in a straightforward way. In Section 3 we formulate and prove the representation theorems under the null hypotheses. Section 4 illustrates how a maximum likelihood estimation situation can fit into our methodology. In Section 5 the results of Section 3 are extended to also cover a sequence of alternatives. Along the way we point out how the results of Durbin follow from ours. In Section 6 the in-probability representation result of Section 3 is extended to the estimated multivariate empirical process.”

Reviewed by *Georg Neuhaus*

Section 5.7 in

Csörgő, M., Révész, P. (1981). *Strong approximations in probability and statistics*. Academic Press, New York, NY,

is based on the 1979 submitted version of the above 1982 paper in *Ann. Probab.* **7**.

MR532241 (80g:60027) 60F15 (60F10 62G20)

**Csörgő, Sándor**

**Erdős-Rényi laws.** *Ann. Statist.* **7** (1979), no. 4, 772-787.

Let  $X_1, X_2, \dots$  be a sequence of nondegenerate, real-valued i.i.d. random variables having a finite moment-generating function on some interval. In their so-called “new law of large numbers”, P. Erdős and A. Rényi [*J. Analyse Math.* **23** (1970), 103-111; MR0272026 (42 #6907)] were able to prove that the maximum averages of blocks of size  $c \log n$  in the set  $\{X_1, \dots, X_n\}$  have an almost sure limit  $\alpha(c)$  as  $n \rightarrow \infty$ , which, as a function of  $c$ , uniquely determines the common distribution function of the  $X_n$ . Recognizing that the proof of the Erdős-Rényi (ER) law does not depend on the special choice of the functional (here average) of the random variables but on the facts that, first, a first order large deviation theorem holds under this functional, and second, it is possible to take sufficiently many independent blocks, the author proves two very general versions of the ER law. The first deals with functionals of i.i.d. random variables taking values in an arbitrary measure space and the second with empirical probability measures of such random variables. In a series of examples, ER laws are derived for often used test statistics and point estimators such as sample quantiles, trimmed means,  $t$ ,  $F$ , chi-square, likelihood ratio and rank statistics, functionals of empirical processes, maximum likelihood estimators, etc. The paper contains an excellent up-to-date survey on the story of the ER law and it also summarizes a great deal of the literature on large deviations and Bahadur slopes of the test statistics and estimators mentioned above.

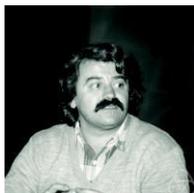
Reviewed by *J. Steinebach* (Düsseldorf)

**Csörgő S**

**BAHADUR EFFICIENCY AND ERDŐS-RÉNYI MAXIMA**

*SANKHYA A* 41: pp. 141-144, (1979)

Concludes that Bahadur slopes of test statistics can be represented by an almost sure limit of the respective E-R maxima of the test statistics in hand.



## EMPIRICAL CHARACTERISTIC FUNCTIONS

$X, X_1, X_2, \dots$  i.i.d.rv's with distribution function  $F$  and characteristic function  $c(t) = \int_{-\infty}^{+\infty} e^{itx} dF(x)$ . Define the empirical characteristic function  $c_n(t)$  of a random sample  $X_1, \dots, X_n$ ,  $n \geq 1$ , on  $X$  by

$$c_n(t) := n^{-1} \sum_{k=1}^n e^{itX_k} = \int_{-\infty}^{+\infty} e^{itx} dF_n(x), \quad -\infty < t < +\infty,$$

where  $F_n(\cdot)$  is the empirical dist<sup>n</sup> function based on the random sample in hand. Define the *empirical characteristic process*  $Y_n(\cdot)$  by

$$\begin{aligned} Y_n(t) &:= n^{1/2}(c_n(t) - c(t)) = \int_{-\infty}^{+\infty} e^{itx} dn^{1/2}(F_n(x) - F(x)) \\ &=: \int_{-\infty}^{\infty} e^{itx} d\beta_n(x), \quad -\infty < t < +\infty. \end{aligned}$$

A. Feuerverger and R.A. Mureika [*Ann. Statist.* **5** (1977), no.1, 88-97] initiated the systematic study of the empirical characteristic function  $c_n(t)$ .

**Csörgő S**

LIMIT BEHAVIOR OF THE EMPIRICAL CHARACTERISTIC FUNCTION *Annals of Probability* 9 pp. 130-144. (1981)

MULTIVARIATE CHARACTERISTIC FUNCTIONS AND TAIL BEHAVIOR *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete* 55: pp. 197-202, (1981)

MULTIVARIATE EMPIRICAL CHARACTERISTIC FUNCTIONS *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete* 55: pp. 203-229, (1981)

### **GLIVENKO-CANTELLI-type THEOREMS:**

$$\|c_n(\cdot) - c(\cdot)\| := \sup_{T_n^{(1)} \leq t \leq T_n^{(2)}} |c_n(t) - c(t)| \rightarrow 0, \quad a.s., \quad n \rightarrow \infty,$$

with

$$\begin{aligned} T_n &:= |T_n^{(1)} \vee T_n^{(2)}| = o((n/\log n)^{1/2}) \quad (\text{Fen-Mu (1977),} \\ &\hspace{15em} \text{provided } F \text{ on } R\dots) \\ &= o((n/\log \log n)^{1/2}) \quad \text{for any } F \text{ on } R \quad (\text{Sándor, AP 1981}) \end{aligned}$$

Csörgő, S - Totik, V (1983): for any  $F$  on  $R^d$ ,  $d \geq 1$ ,

$$\sup_{|t| \leq T_n} |c_n(t) - c(t)| \rightarrow 0, \quad a.s., \quad n \rightarrow \infty,$$

if  $\lim_{n \rightarrow \infty} \log T_n/n = 0$ .

**Sándor Csörgő** (SCs), in the just mentioned AP 1981 paper: All finite dimensional distributions of  $Y_n(\cdot)$  converge to those of the complex Gaussian process

$$Y(t) = \int_{-\infty}^{+\infty} e^{itx} dB(F(x)),$$

that may however have discontinuous sample functions a.s. if  $F$  has only low logarithmic moments, and hence weak convergence of  $Y_n$  to  $Y$  *necessarily* fails for such  $F$ ;  $B(\cdot)$  is a Brownian bridge.

Necessary and sufficient conditions are given for the a.s. continuity of  $Y$  and, assuming that  $E|X|^\alpha = \int |x|^\alpha dF(x) < \infty$  for some  $\alpha > 0$ , then, e.g., via KMT (1975), with  $r \leq s$  fixed:

$$\|Y_n(\cdot) - Z_n(\cdot)\|_r^s \stackrel{a.s.}{=} O\left(n^{-\alpha/(2\alpha+4)}(\log n)^{(\alpha+1)/(\alpha+2)}\right).$$

where  $Z_n(\cdot) \stackrel{D}{=} Y(\cdot)$  for each  $n \geq 1$ .

M.B. Marcus [Ann., Probab. **9** (1981), 194-201] has subsequently shown that the necessary and sufficient conditions as in SCs: AP 1981 for sample continuity of  $Y(\cdot)$  are *also* (necessary and) *sufficient* for the weak convergence of  $Y_n$  to  $Y$ , as conjectured in SCs: AP 1981.

MR785384 (86e:62071) 62G30

Csörgő, Sándor (H-SZEG-B)

**Testing by the empirical characteristic function: a survey.**

*Asymptotic statistics*, 2 (Kutná Hora,1983),45-56, Elsevier, Amsterdam, 1984.

Author summary “Many distributional properties are either more conveniently characterized through the characteristic function than through the distribution or density functions, or characterized only through the characteristic function. This suggests that such properties should or could be tested either more conveniently, or solely, through the use of empirical characteristic functions rather than empirical distributions or densities. In the last few years various large sample testing procedures have been proposed which are based on the asymptotic behaviour of univariate or multivariate empirical characteristic functions. These tests are surveyed here.”

{For the entire collection see MR0785381 (86d:62003)}

## Random Censorship Models

Let  $X_1, X_2, \dots, X_n$ , be independent rv's (*survival times*) with distribution function  $F(x) = P(X \leq x), x \in \mathbb{R}$ . An independent sequence of independent rv's  $Y_1, Y_2, \dots, Y_n$  with distribution function  $G$  *censors* them on the right so that one can only observe

$$Z_j = \min(X_j, Y_j) \quad \text{and} \quad \delta_j = I\{X_j \leq Y_j\}, \quad j = 1, \dots, n,$$

i.e., **one observes the  $n$  pairs**  $(Z_j, \delta_j), 1 \leq j \leq n$ , where

$$Z_j = X_j \wedge Y_j \quad \text{and} \quad \delta_j = \begin{cases} 1 & \text{if } X_j \leq Y_j \text{ (} X_j \text{ is uncensored)} \\ 0 & \text{if } X_j > Y_j \text{ (} X_j \text{ is censored).} \end{cases}$$

Thus the  $Z_j, 1 \leq j \leq n$ , are i.i.d. rv's with distribution function  $H$  given by  $1 - H = (1 - F)(1 - G)$ ; a *useful model* for a variety of problems *in biostatistics and life testing (survival analysis)*. In case of  $G$  *degenerate*, this model *reduces to the fixed censorship model*.

Let  $Z_{1,n} \leq \dots \leq Z_{n,n}$  denote the order statistics of  $Z_1, \dots, Z_n$  with the corresponding concomitants  $\delta_{1,n}, \dots, \delta_{n,n}$  so that  $\delta_{j,n} = \delta_i$  if  $Z_{j,n} = Z_i$ . For a *continuous*  $F$  in this *random censorship model* the Kaplan-Meier product-limit estimator (PL) (E.L. Kaplan and P. Meier, JASA **53** (1958), 457-481, 41551 *citations*) of the survival function  $S := 1 - F$  is defined as

$$S_n(x) = 1 - \hat{F}_n(x) := \prod_{j=1}^n \left( 1 - \frac{\delta_{j,n}}{n - j + 1} \right)^{I(Z_{j,n} \leq x)}, \quad x \in \mathbb{R}.$$

$\hat{F}_n$  is the nonparametric maximum likelihood estimator of  $F$  based on  $\{(Z_i, \delta_i), 1 \leq i \leq n\}$  (cf. Johansen, S. (1978). The product limit estimator as a maximum likelihood estimator, *Scandinavian Journal of Statistics* **5**, 195-199).

For further related work on  $\hat{F}_n$  in this regard and what the authors call their “nonparametric Cox model”, we refer to

Major, P. and Rejtő, L. (1998). A note on nonparametric estimations, pp. 759-774,

and

Rejtő, L. and Tusnády, G. (1998). On the Cox regression, pp. 621-637,

BOTH papers in *Asymptotic Methods in Probability and Statistics*, B. Szyszkowicz (Editor), 1998 Elsevier Science B.V.

Several papers have dealt with the problem of **strong consistency of the PL estimator**. In particular, we mention:

Földes, A. and Rejtő, L. (1981). Strong uniform consistency for nonparametric survival curve estimators from randomly censored data. *Ann. Statist.* **9**, 122-129.

Földes, A. and Rejtő, L. (1981). A LIL type result for the product limit estimator. *Z. Wahrsch. verw. Gebiete* **56**, 75-86.

Csörgő, S and Horváth, L. (1983). The rate of strong uniform consistency for the product-limit estimator. *Z. Wahrsch. Verw. Gebiete* **62**, 411-426.

**The first strong approximation result for the PL estimator process:**

Burke, M.D., Csörgő, S, Horváth L.

STRONG APPROXIMATIONS OF SOME BIOMETRIC ESTIMATES UNDER  
RANDOM CENSORSHIP

*ZEITSCHRIFT FUR WAHRSCHEINLICHKEITSTHEORIE UND VERWANDTE  
GEBIETE* 56: pp. 87-112. (1981)

For the mentioned *random censorship model* let  $S_n := 1 - \hat{F}_n$  be the Kaplan-Meier estimator of the survival function  $S = 1 - F$ , assumed to be continuous. The cumulative hazard is given as  $\Lambda = -\log S$  and estimated by  $\Lambda_n := -\int_{-\infty}^t S_n^{-1}(s) dS_n$ . These notions generalize to that of the multiple decrements (*competing risks*) model that is studied in this paper. In particular, strong approximations are studied for the processes  $n^{1/2}(S_n(t) - S(t))$ ,  $n^{1/2}(\Lambda_n(t) - \Lambda(t))$  and  $n^{1/2}(\exp(-\Lambda_n(t)) - S(t))$ ,  $t \in \mathbb{R}$ .

The aim of the respective strong approximations of these processes by appropriate Gaussian processes is to see how far one can go uniformly in  $t \in \mathbb{R}$ , and at what rate in  $n$ , as  $n \rightarrow \infty$ . In the paper in hand, for each  $n \geq 1$ ,  $T_n = T_n(\varepsilon)$  is a number such that  $T_n < \inf\{t : H(t) = 1\}$  and  $1 - H(T_n) \geq (2\varepsilon n^{-1} \log n)^{1/2}$ , where  $H$  is the distribution function of  $Z_1 = \min\{X_1, Y_1\}$ , and  $\varepsilon$  is any given positive number.

**Burke, M.D., Csörgő, S, Horváth L.**

A CORRECTION TO AND IMPROVEMENT OF “STRONG APPROXIMATIONS OF SOME BIOMETRIC ESTIMATES UNDER RANDOM CENSORSHIP”. *PROBABILITY THEORY AND RELATED FIELDS* 79: pp. 51-57. (1988)

**Re their 1981 ZfW paper the authors write:**

There is an error in the proof of the main result in [1]. The aim of this note is to correct the error using a recent inequality of Dehling, Denker and Philipp [2]. In this way we in fact improve all the approximation rates claimed in [1]. In the interest of saving space we formulate and prove the new results in the Kaplan-Meier model instead of the greater generality of the competing risks model used in[1]. The results, however, hold true in that generality.

**MR952993 (89h:62058)** 62G05 (62P10)

**Burke, Murray D., Csörgő, Sándor, Horváth Lajos**

A CORRECTION TO AND IMPROVEMENT OF “STRONG APPROXIMATIONS OF SOME BIOMETRIC ESTIMATES UNDER RANDOM CENSORSHIP”. [*Z. Wahrsch. verw. Gebiete* **56** (1981), no.1, 87-112; **MR0612162 (83a:62102)**].

*Probab. Theory Related Fields* **79** (1988), no. 1, 51-57.

In the paper cited in the heading there is an error. Using a recent inequality of H. Dehling, M. Denker and W. Philipp [*Ann. Inst. H. Poincaré* **23**(1987), no.2, 121-134; **MR0891707 88i:60061**], the authors not only correct the proof, but also improve the result so that the rates of approximation of the limiting Gaussian processes reduce to the rates of J. Komlós, P. Major and G. Tusnády [*Z. Wahrsch. Verw. Gebiete* **32**, 111-131; **MR0375412 (51#11605b)**] for the uncensored empirical process.

Reviewed by *Niels Keiding*

Inspired by the B-SCs-H (1981) first strong approximation for the PL *estimator process*

$$n^{1/2}(S_n - S) := n^{1/2}(F - \hat{F}_n),$$

Major, P. and Rejtő, L. Strong embedding of the estimator of the distribution function under random censorship. *Ann. Statist.* **16**(1988), no. 3, 1113-1132,

with  $T$  such that  $1 - H(T) > \delta$  with some  $\delta > 0$ , also improve the rate of the B-SCs-H (1981) approximation of the PL *estimator process* by appropriate Gaussian processes so that the rates reduce to those of KMT (1975).

Inspired by B-SCs-H(1981), and based on preliminary versions of

E.-E. Aly, M. Csörgő and L. Horváth, Strong approximations of the quantile process of the product-limit estimator. *Journal of Multivariate Analysis* **16**(1985), no.2, 185-210,

in Chapter 8 of

M. Csörgő, *Quantile processes with statistical applications*. CBMS-NSF Regional Conference Series in Applied Mathematics **42**, SIAM Philadelphia 1983,

the PL and PL-quantile processes are approximated simultaneously by the same Kiefer-type Gaussian processes at the a.s. rate  $O(n^{-1/4}(\log n)^{1/2}(\log \log n)^{1/4})$ , with  $T$  appropriately fixed.



## Csörgő S

UNIVERSAL GAUSSIAN APPROXIMATIONS UNDER RANDOM CENSORSHIP *Annals of Statistics* **24**; pp. 2744-2778 (1996).

Immersed in 46 references, this paper by Sándor is **likely the ultimate one** on approximations under random censorship in that they hold uniformly up to some large order statistics in the sample  $Z_i = \min\{X_i, Y_i\}$ ,  $1 \leq i \leq n$ , with optimal approximation rates depending on the order of these statistics.

Inspired, and sorting out problems posed, by the papers

**Gill, R.,D.** (1983). Large sample behavior of the product-limit estimator on the whole line. *Ann. Statist.* **11** 49-58,

**Stute, W.** (1994). Strong and weak representations of cumulative hazard functions and Kaplan-Meier estimators on increasing sets. *J. Statist. Plann. Inference* **42** 315-329,

**SCs** (1996) returns to the primary problem of Gaussian approximations in BSCsH(1981, 1988), posing and answering the question of how far out do Gaussian approximations hold universally under random censorship.

For example with integers  $1 \leq k_n < n$ , one of the **nine optimal approximations** of Theorem 2 concludes: on a suitable probability space there exist a sequence  $\{W_n(\cdot)\}$  of standard Wiener processes so that

$$(a) \quad \sup_{x \leq Z_{n-k_n, n}} \left| \sqrt{n} \frac{\hat{F}_n(x) - F(x)}{1 - \hat{F}_n(x)} - W_n(d(x)) \right| = O_P \left( \frac{\sqrt{n} \log n}{k_n} \right),$$

where  $d(x)$  is the asymptotic variance of the Kaplan-Meier PL-estimator  $\hat{F}_n$  of  $F$ .

Also, with  $D(x) := \frac{d(x)}{1 + d(x)}$  and  $d_n(x)$  as the estimated counterpart of the asymptotic variance  $d(x)$  of the Kaplan-Meier PL-estimator  $\hat{F}_n$  of *continuous*  $F$ , with integers  $1 \leq k_n < n$ , another conclusion of Theorem 2 reads:

$$(b) \quad \sup_{x \leq Z_{n-k_n, n}} \left| \frac{\sqrt{n}(\hat{F}_n(x) - F(x))}{(1 - \hat{F}_n(x))(1 + d_n(x))} - B_n(D(x)) \right| \\ = O_P \left( \frac{n}{k_n^{3/2}} + \frac{\sqrt{n} \log n}{k_n} \right)$$

with an appropriately constructed sequence of Brownian bridges on a suitable probability space.

Both (a) and (b) deal with empirical processes *à la*

**Rényi, A** (1953). On the theory of order statistics. *Acta Math. Acad. Sci. Hungar.* **4** 191-231.

Theorem 1 deals with the estimated cumulative hazard function  $\Lambda_n$  in a similar vein in **six optimal approximations**.

**MR680636 (84c:62063) 62G10 (2G05)**

**Csörgő, Sándor; Horváth, Lajos**

Statistical inference from censored samples. (Hungarian. English summary) *Alkalmaz. Mat. Lapok* **8**(1982), no.1-2, 1-89.

This paper is an up-to-date and fairly complete survey of the field of statistical inferences from censored data. Motivated by medical applications, several known results are improved; thus, they are much more applicable for finite sample size. Unfortunately, since the paper is written in Hungarian, it may not be accessible to many statisticians.

Reviewed by *László Györfi*

This is a most impressive 89 page survey of the area in hand, reaching back to Daniel Bernoulli, who posed the first problem concerning inference from censored samples in 1760. Based on, and in addition to, their own series of articles at that time [47, 52-60, 101-104], there are 173 papers referenced, providing an overview of the most frequently used non-parametric random censorship models and methods up to that time. In his just mentioned *Annals. Stat.* (1996) paper, Sándor revisits and provides improvements to both papers that are mentioned herewith. The above one in Hungarian continues to be highly desirable to be translated and republished.

**MR849868 (87i:62089) 62G15 (62E20 62N05)**

**Csörgő, Sándor (H-SZEG-B); Horváth, Lajos (H-SZEG-B)**

Confidence bands from censored samples. (French summary)

*Canad. J. Statist.* **14** (1986), no 2, 131-144.

... Several different methods for constructing such bands are described and compared. The methods are illustrated using survival data for pacemaker patients.

Reviewed by *Lionel Weiss*

# Lecture Notes in Statistics

Edited by D. Brillinger, S. Fienberg, J. Gani,  
J. Hartigan, and K. Krickeberg

33

Miklós Csörgő  
Sándor Csörgő  
Lajos Horváth

An Asymptotic Theory  
for Empirical Reliability  
and Concentration Processes



Miklós Csörgö

Sándor Csörgö

Lajos Horváth

David M. Mason

An Asymptotic Theory for Empirical Reliability and  
Concentration Processes

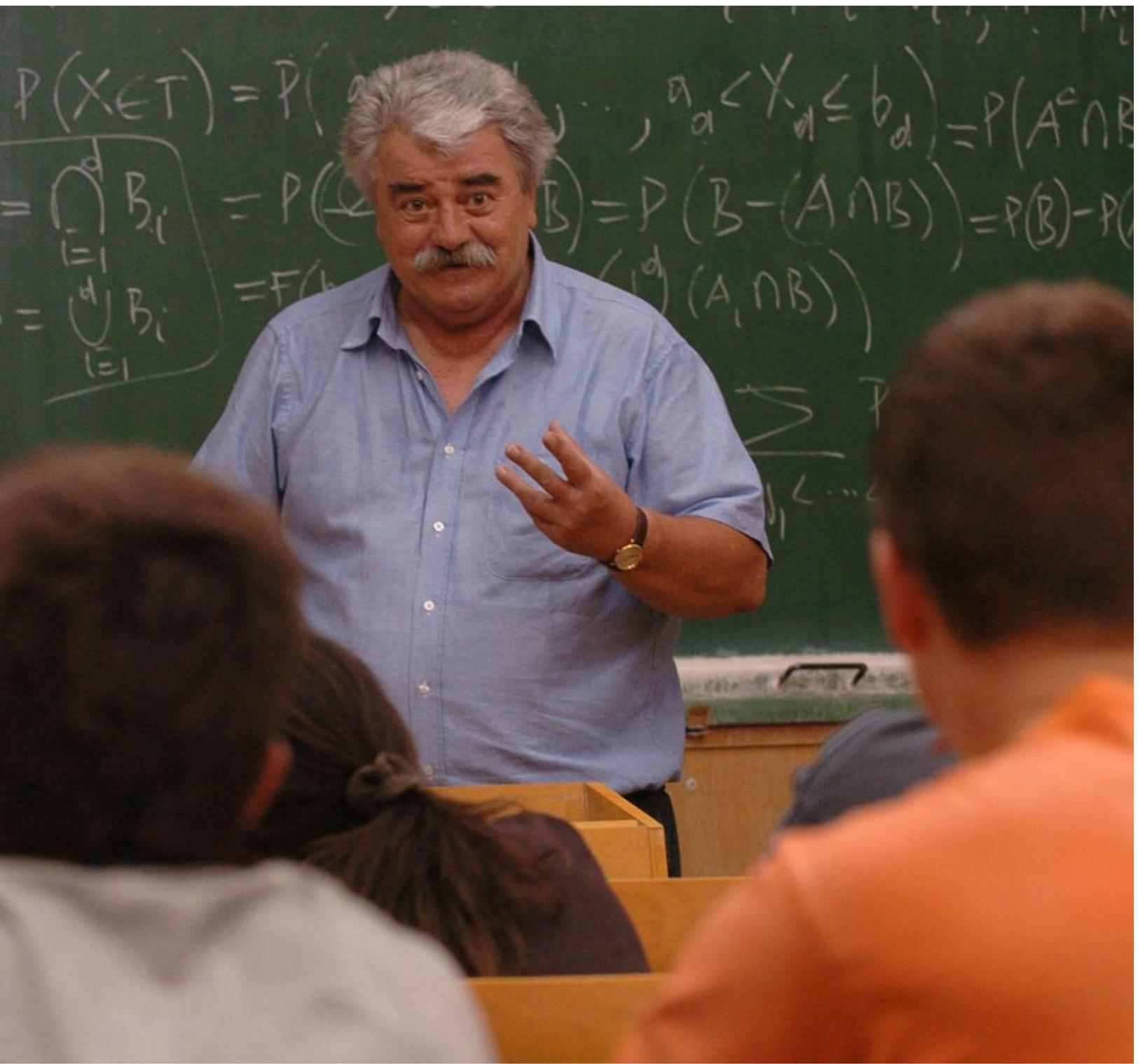
**PREFACE** Miklós Csörgő and David M. Mason initiated their collaboration on the topics of this book while attending the CBMS-NSF Regional Conference at Texas A & M University in 1981. Independently of them, Sándor Csörgő and Lajos Horváth have begun their work on this subject at Szeged University. The idea of writing a monograph together was born when the four of us met in the Conference on Limit Theorems in Probability and Statistics, Veszprém 1982. This collaboration resulted in No.2 of Technical Report Series of the Laboratory for Research in Statistics and Probability at Carleton University and University of Ottawa, 1983. Afterwards David M. Mason has decided to withdraw from this project. The authors wish to thank him for his contributions. In particular, ... These and several other related remarks helped us push down the moment condition to  $EX^2 < \infty$  in all our weak approximation theorems.

Readership: Statistician, reliability theorist, economist, biometrician

The 'total time on test' is used in reliability engineering, the 'Lorenz curve' in economics, and 'mean residual life' in biostatistics. Here is a unified treatment of the asymptotics of all these, based on strong approximation of stochastic processes and the authors' powerful quantile methods. Processes relating to the above statistics involve sums of order statistics in some form, and other such processes are introduced for specific purposes. Approximating processes are Gaussian. The treatment is clear and thorough and quite concrete, and the asymptotics of many specific functionals are worked out. Estimation procedures including bootstrap methods are also considered.

University of Sussex  
Brighton, U.K.

C.M. Goldie





**Weighted empirical and quantile processes & normal and stable convergence of integral functions of the empirical distribution function.**

114 Csörgő M, Csörgő S, Horváth L, Mason D M

WEIGHTED EMPIRICAL AND QUANTILE PROCESSES  
**ANNALS OF PROBABILITY** 14: pp. 31-85. (1986)

116 Csörgő M, Csörgő S, Horváth L, Mason D M

NORMAL AND STABLE CONVERGENCE OF INTEGRAL FUNCTIONS OF THE EMPIRICAL DISTRIBUTION FUNCTION  
**ANNALS OF PROBABILITY** 14: pp. 86-118. (1986)

Let  $U_1, U_2, \dots$  be independent uniformly distributed random variables on the unit interval  $[0, 1]$ , and let  $U_{1,n} \leq \dots \leq U_{n,n}$  be their first  $n \geq 1$  order statistics. The uniform empirical quantile function is defined by

$$U_n(s) = U_{k,n}, \quad (k-1)/n < s \leq k/n \quad (k = 1, \dots, n),$$

where  $U_n(0) = U_{1,n}$ , and the uniform quantile process is

$$u_n(s) = n^{1/2}(s - U_n(s)), \quad 0 \leq s \leq 1.$$

The corresponding uniform empirical distribution function is

$$G_n(s) = n^{-1} \#\{1 \leq i \leq n, U_i \leq s\}$$

and

$$\alpha_n(s) = n^{1/2}(G_n(s) - s), \quad 0 \leq s \leq 1.$$

is the uniform empirical process. One of the results in [114] is the following theorem.

**Theorem** (MCs-SCs-H-M (1986)) *The probability space of  $u_n$  and  $\alpha_n$  can be so extended that with a sequence of Brownian bridges  $\{B_n(s), 0 \leq s \leq 1\}$  on it we have*

$$\sup_{\lambda/n \leq s \leq 1 - \lambda/n} n^\nu |u_n(s) - B_n(s)| / (s(1-s))^{1/2-\nu} = O_P(1) \quad (1)$$

for every  $0 \leq \nu < \frac{1}{2}$ , and

$$\sup_{\lambda/n \leq s \leq 1 - \lambda/n} n^\tau |\alpha_n(s) - B_n(s)| / (s(1-s))^{1/2-\tau} = O_P(1) \quad (2)$$

for every  $0 \leq \tau < \frac{1}{4}$ , as  $n \rightarrow \infty$ , for all  $0 < \lambda < \infty$ .

For the same construction we also have

$$\sup_{0 \leq s \leq 1} |\alpha_n(s) - B_n(s)| = O(n^{-1/4}(\log n)^{1/2}(\log \log n)^{1/4}) \text{ a.s.}, \quad (3)$$

$$\sup_{0 \leq s \leq 1} |u_n(s) - B_n(s)| = O(n^{-1/2}(\log n)) \text{ a.s.}, \quad (4)$$

Statement (4) was first established in

Csörgő, M. and Révész, P. (1975). Some notes on the empirical distribution function and the quantile process. *Limit Theorems of Probability Theory* **11** 59-71 (Colloquia Mathematica Societatis János Bolyai, Keszthely, Hungary). North Holland, Amsterdam.

Cf. also MCs-R (1978), *Ann. Statist.* **6**, 882-894.

For (1), (2), (3), (4) “*the other way around*”, cf. Mason and van Zwet (1987), *Ann. Probab.* **15** 871-884.

For a **bootstrap parallel** of the weighted approximations as in (1) & (2), consult

**Csörgő S, Mason D M**

BOOTSTRAPPING EMPIRICAL FUNCTIONS

**ANNALS OF STATISTICS** 17: pp. 1447-1471 (1989)

and for that of (3) with the KMT (1975)  $O(n^{-1/2} \log n)$  a.s. rate, we refer to

Csörgő, M., Horváth, L. and Kokoszka, P. (2000)

*Proceedings of the American Mathematical Society* **128**, 2457-2464.

Inspiring works along these lines

**Csörgő, S**

ON THE LAW OF LARGE NUMBERS FOR THE BOOTSTRAP MEAN

**STATISTICS & PROBABILITY LETTERS** 14: PP. 1-7. (1992)

**Csörgő, S and Rosalsky, A**

A SURVEY OF LIMIT LAWS FOR BOOTSTRAPPED SUMS

**INTERNATIONAL JOURNAL OF MATHEMATICS AND**

**MATHEMATICAL SCIENCES** 45: pp. 2835-2861. (2003)

the latter with 78 references.

# Another look at Bootstrapping the Student $t$ -statistic

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<http://arxiv.org/abs/1209.4089v4>

*Dedicated to the memory of Sándor Csörgő*

## Abstract

Let  $X, X_1, X_2, \dots$  be a sequence of i.i.d. random variables with mean  $\mu = EX$ . Let  $\{v_1^{(n)}, \dots, v_n^{(n)}\}_{n=1}^\infty$  be vectors of non-negative random variables (weights), independent of the data sequence  $\{X_1, \dots, X_n\}_{n=1}^\infty$ , and put  $m_n = \sum_{i=1}^n v_i^{(n)}$ . Consider  $X_1^*, \dots, X_{m_n}^*$ ,  $m_n \geq 1$ , a bootstrap sample, resulting from *re-sampling* or *stochastically re-weighing* a random sample  $X_1, \dots, X_n$ ,  $n \geq 1$ . Put  $\bar{X}_n = \sum_{i=1}^n X_i/n$ , the original sample mean, and define  $\bar{X}_{m_n}^* = \sum_{i=1}^{m_n} v_i^{(n)} X_i/m_n$ , the bootstrap sample mean. Thus,  $\bar{X}_{m_n}^* - \bar{X}_n = \sum_{i=1}^n (v_i^{(n)}/m_n - 1/n) X_i$ . Put  $V_n^2 = \sum_{i=1}^n (v_i^{(n)}/m_n - 1/n)^2$  and let  $S_n^2, S_{m_n}^{*2}$  respectively be the the original sample variance and the bootstrap sample variance. The main aim of this exposition is to study the asymptotic behavior of the bootstrapped  $t$ -statistics  $T_{m_n}^* := (\bar{X}_{m_n}^* - \bar{X}_n)/(S_n V_n)$  and  $T_{m_n}^{**} := \sqrt{m_n}(\bar{X}_{m_n}^* - \bar{X}_n)/S_{m_n}^*$  in terms of *conditioning on the weights* via assuming that, as  $n, m_n \rightarrow \infty$ ,  $\max_{1 \leq i \leq n} (v_i^{(n)}/m_n - 1/n)^2/V_n^2 = o(1)$  almost surely or in probability on the probability space of the weights. In consequence of these maximum negligibility conditions of the weights, a characterization of the validity of this approach to the bootstrap is obtained as a direct consequence of the Lindeberg-Feller central limit theorem. This view of justifying the validity of the bootstrap of i.i.d. observables is believed to be new. The need for it arises naturally in practice when exploring the nature of information contained in a random sample via re-sampling, for example. Unlike in the theory of weighted bootstrap with exchangeable weights, in this exposition it is not assumed that the components of the vectors of non-negative weights are exchangeable random variables. *Conditioning on the data* is also revisited for Efron's bootstrap weights under conditions on  $n, m_n$  as  $n \rightarrow \infty$  that differ from requiring  $m_n/n$  to be in the interval  $(\lambda_1, \lambda_2)$  with  $0 < \lambda_1 < \lambda_2 < \infty$  as in Mason and Shao (2001). Also, the validity of the bootstrapped  $t$ -intervals is established for both approaches to conditioning. Moreover, when conditioning on the sample, our results in this regard are new in that they are shown to hold true when  $X$  is in the domain of attraction of the normal law (DAN), possibly with infinite variance, while the ones for  $EX^2 < \infty$  when conditioning on the weights are first time results *per se*.

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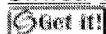
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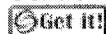
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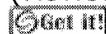
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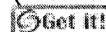
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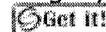
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13:30-14:00

## Kernel Estimators of the Tail Index

DAVID M. MASON\*†

\*Department of Applied Economics and Statistics, University of Delaware, Newark, Delaware, USA

†email: davidm@udel.edu

My talk is partially based on joint work with Sándor Csörgő, Paul Deheuvels and Julia Dony. It will present two interesting applications of the technologies of weighted approximations and modern empirical process theory, and will provide me with the opportunity to discuss the general lines of my collaboration with my longtime friend Sándor.

*Acknowledgment.* Part of my research was supported by an NSF Grant.

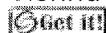
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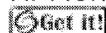
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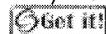
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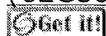


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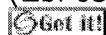


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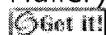
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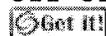
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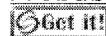
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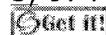
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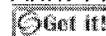
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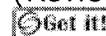
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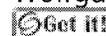
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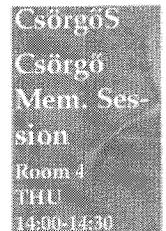


## Merging in Generalized St. Petersburg Games

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\*MTA–SZTE Analysis and Stochastics Research Group, Bolyai Institute, Szeged, Hungary

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In this talk I present some important contributions of Sándor Csörgő to one of his favourite problems, to the St. Petersburg game.

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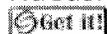
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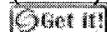
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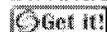
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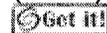
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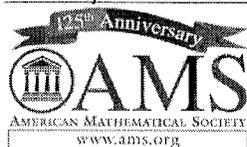


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$$F = P(X \in T) \geq 0$$

$$B_i = \{X_i \leq b_i\}, A_i = \{X_i \leq a_i\}$$

$$(X \in T) =$$



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THU  
14:30-15:00

## The $m(n)$ out of $k(n)$ Bootstrap for Partial Sums of St. Petersburg Type Games

EUSTASIO DEL BARRIO\*, ARNOLD JANSSEN<sup>†‡</sup>, MARKUS PAULY<sup>†</sup>

\*Universidad de Valladolid, Facultad de Ciencias, C/ Prado de la Magdalena s/n, Spain,

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As a concrete example we study bootstrap limit laws for the cumulated gain sequence of repeated St. Petersburg games. For these games the investigation of distributional convergent partial sums have e.g. been investigated by Martin-Löf (1985), Csörgő and Dodunekova (1991), Csörgő (2010) and Gut (2010). Here it is shown that the bootstrap inherits these partial limit laws. In particular, a continuum of different semi-stable bootstrap limit laws occur for classical and generalized St. Petersburg games.

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MR2668419 (2011g:60039) 60F05 (60F99)

Csörgő, Sándor

**Probabilistic approach to limit theorems for the St. Petersburg game.**

With a preface by Vilmos Totik.

*Acta Sci. Math. (Szeged)* **76** (2010), no. 1-2, 233–350.

This paper is concerned with the probabilistic aspect of the celebrated St. Petersburg game. This is a thorough analysis developed by the late Sándor Csörgő. The paper is a valuable source of numerous limit theorems related to the game. In particular, an asymptotic distribution of sums of order statistics is established.

Reviewed by *Anna Jaśkiewicz*

49 **References**

Sándor Csörgő  
Gordon Simons

A BIBLIOGRAPHY OF THE  
ST. PETERSBURG PARADOX

## Introduction

Peter tosses a fair coin repeatedly until it first lands heads and pays Paul  $2^k$  ducats if this happens on the  $k$ -th toss,  $k = 1, 2, \dots$ . What is a fair price for Paul to pay to Peter for the game? It is an infinite number of ducats but, as Nicolas Bernoulli wrote, "... any fairly reasonable man would sell his chance, with great pleasure, for forty ducats". This is the Petersburg paradox. (In the original formulation of the problem, Paul's gain was half of the above. Following Feller (1950), for the sake of simplicity, we have doubled the gain and have taken the associated liberty of doubling the figure in the citation: Nicolas said "twenty".)

A variant of the problem, the same reward system for obtaining the first "six" on a die, was first proposed in 1713 by Nicolas Bernoulli (1687-1759), nephew of both the famous brothers Jakob (1654-1705) and Johann (1667-1748) Bernoulli, in a letter to de Montmort, and was published the same year in the second edition of de Montmort's book. The above formulation of the problem for coin tossing was suggested by another Swiss mathematician, Gabriel Cramer (1704-1752), a student of Johann's, in a letter to Nicolas in 1728, who in the meantime also induced his cousin Daniel Bernoulli (1700-1782), son of Johann, to think about the paradox. Daniel at the time worked in St. Petersburg, Russia, and the wider scientific community learned about the problem from his famous essay "Specimen theoriae novae de mensura sortis" submitted in 1731 to and published in 1738 in the *Commentarii Acad. Sci. Imp. Petropolitanae*. Hence the name of the paradox, coined by d'Alembert. Part of the correspondence among the various Bernoullis and Cramer was reproduced in Daniel's essay; the citation above is taken from there. (However, to save space, thereby resisting what SAMUELSON (1977) calls a "seduction by the antiquarian charms of the problem", we refer to the historical sources only indirectly through four recent longer historical surveys, cited two paragraphs below.)

Almost every leading thinker of some mathematical note in the eighteenth century entered into the discussion of the paradox. A large number of different resolutions have been proposed, each one becoming the immediate target of vehement criticism. Part of the difficulty was that the notion of expectation, presently in-

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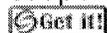
Mathematical Reviews

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**MR2448396** Reviewed [Csörgő, Sándor](#); [Simons, Gordon](#) Weak laws of large numbers for cooperative gamblers. *Period. Math. Hungar.* 57 (2008), no. 1, 31–60. [65C50](#)



**MR2392789** Reviewed [Csörgő, Sándor](#); [Simons, Gordon](#) St. Petersburg games with the largest gains withheld. *Statist. Probab. Lett.* 77 (2007), no. 12, 1185–1189. [62E15](#) ([60F15](#) [62G30](#))



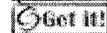
**MR2274852** Reviewed [Csörgő, Sándor](#); [Simons, Gordon](#) Pooling strategies for St. Petersburg gamblers. *Bernoulli* 12 (2006), no. 6, 971–1002. (Reviewer: Oliver Johnson) [62C15](#) ([62B10](#) [91A60](#))



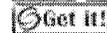
**MR2162802** Reviewed [Csörgő, Sándor](#); [Simons, Gordon](#) Laws of large numbers for cooperative St. Petersburg gamblers. *Period. Math. Hungar.* 50 (2005), no. 1-2, 99–115. (Reviewer: Oliver Johnson) [60F05](#) ([94A17](#))



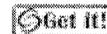
**MR1979976** Reviewed [Csörgő, S.](#); [Simons, G.](#) The two-Paul paradox and the comparison of infinite expectations. *Limit theorems in probability and statistics, Vol. I (Balatonlelle, 1999)*, 427–455, *János Bolyai Math. Soc., Budapest*, 2002. (Reviewer: R. A. Maller) [60F05](#) ([60E05](#) [62G30](#))



**MR1385664** Reviewed [Csörgő, Sándor](#); [Simons, Gordon](#) A strong law of large numbers for trimmed sums, with applications to generalized St. Petersburg games. *Statist. Probab. Lett.* 26 (1996), no. 1, 65–73. (Reviewer: R. A. Maller) [60F15](#) ([62G30](#))



**MR1359832** Reviewed [Csörgő, Sándor](#); [Simons, Gordon](#) Precision calculation of distributions for trimmed sums. *Ann. Appl. Probab.* 5 (1995), no. 3, 854–873. (Reviewer: Erich Haeusler) [60E99](#) ([60-04](#) [60F05](#))



**MR1321758** Reviewed [Csörgő, Sándor](#); [Simons, Gordon](#) On Steinhaus' resolution of the St. Petersburg paradox. *Probab. Math. Statist.* 14 (1993), no. 2, 157–172 (1994). (Reviewer: Lars Holst) [60C05](#) ([60F99](#))

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Peter tosses a fair coin repeatedly until it first lands heads and pays Paul  $2^k$  ducats if it happens on the  $k$ -th toss,  $k = 1, 2, \dots$ . What is a fair price for Paul to pay to Peter for the game? Let  $X$  denote Paul's gain in the game, i.e.,  $X$  has possible values  $2^1, 2^2, 2^3, \dots$  with corresponding probabilities  $2^{-1}, 2^{-2}, 2^{-3}, \dots$ . Thus

$$(1) \quad P(X = 2^k) = 2^{-k}, \quad k = 1, 2, \dots, \quad \text{and} \quad EX = \infty.$$

Consider a sequence of independent repetitions of the game, and let  $X_1, X_2, \dots$  be Paul's gains in the first, second, ... Petersburg games, i.e., independent copies of  $X$  in (1). Then  $S_n = X_1 + \dots + X_n$  denotes Paul's total gain in  $n \geq 1$  games. If  $EX$  were finite, then the LLN would imply that the fair price for  $n$  games would be  $nEX$ , for then  $S_n/(nEX) \rightarrow 1$  a.s. as  $n \rightarrow \infty$ . Presently, for Paul's gain we have  $S_n/n \rightarrow \infty$  a.s. as  $n \rightarrow \infty$ .

Feller (1945):

$$(2) \quad S_n/(n \log_2 n) \rightarrow 1 \quad \text{in probability as } n \rightarrow \infty.$$

...

MR1718347 (2001f:60033) 60F15 (60F05 60G50)

Berkes, István(H-AOS); Csáki, Endre(H-AOS); Csörgő, Sándor(1-MI-S)  
ALMOST SURE LIMIT THEOREMS FOR THE ST. PETERSBURG  
GAME. (English summary)

*Statist. Probab. Lett.* 45 (1999), no. 1, 23-30.

The authors consider the following St. Petersburg game: A fair coin is tossed until the first head occurs. If this happens at the  $k$ th trial a player receives  $2^k$  ducats. Repeat this game and consider the sum  $S_n$  and the maximum  $M_n$  of the first  $n$  gains. The distributional behaviour of  $S_n$  is unpleasant. The subsequential limit distributions of  $S_n/n - \log_2 n$  consist of a whole family  $\mathcal{G} = \{G_\gamma, 1/2 \leq \gamma \leq 1\}$  of some specific distributions  $G_\gamma$  [see S. Csörgő and R. D. Dodunekova, in *Sums, trimmed sums and extremes*, 285-315, Birkhäuser Boston, Boston, MA, 1991; MR1117274 (92h:60031)]. However, as is shown in this paper, the logarithmic average shows the following a.s. limiting behaviour:

$$\frac{1}{\log n} \sum_{k=1}^n \frac{1}{k} I\left(\frac{S_k}{k} - \log_2 k \leq x\right) \xrightarrow{a.s.} \frac{1}{\log 2} \int_{1/2}^1 \frac{G_\gamma(x)}{\gamma} d\gamma, \quad n \rightarrow \infty.$$

The proof is based on an approximation of  $P(S_k/k - \log_2 k \leq x)$  by a suitable  $G_{\gamma_k}(x)$  [see S. Csörgő, *Polygon* 5 (1995), no. 1, 1979; per bibl.]. A related result for  $I(M_k/k \leq x)$  is presented as well.

Reviewed by *Ulrich Stadtmüller*

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## Almost sure limit theorems

Berkes I, Csáki E, Csörgő, S

ALMOST SURE LIMIT THEOREMS FOR THE ST. PETERSBURG GAME, **STATISTICS & PROBABILITY LETTERS** 45: pp. 23-30. (1999)

**Abstract.** We show that the accumulated gain  $S_n$  and the maximal gain  $M_n$  in  $n$  St. Petersburg games satisfy almost sure limit theorems with nondegenerate limits, even though ordinary asymptotic distributions do not exist for  $S_n$  and  $M_n$  with any numerical centering and norming sequences.

Berkes I, Csáki E, Csörgő S, Megyesi Z

ALMOST SURE LIMIT THEOREMS FOR SUMS AND MAXIMA FROM THE DOMAIN OF GEOMETRIC PARTIAL ATTRACTION OF SEMISTABLE LAWS

In: Berkes I, Csáki E, Csörgő M (ed.)

Limit Theorems in Probability and Statistics 1-2.

Konferencia helye, ideje: Balatonlelle, Hungary, 28/06/1999-02/07/1999.

Bolyai János Mathematical Society, 2002, pp. 133-157.

(ISBN:963-9453-01-3)

From the **Abstract**

The aim of this paper is to show that sums and maxima from the domain of geometric partial attraction of a semistable law satisfy almost sure limit theorems along the whole sequence  $\{n\} = \mathbb{N}$  of natural numbers, despite the fact that ordinary convergence in distribution typically takes place in both cases only along  $\{k_n\}$  and related subsequences. We describe the class of all possible almost sure asymptotic distributions both for sums and maxima.

**Matches:** 159[Show first 100 results](#)

Publications results for "Items authored by Csörgő, Sándor "

**MR2668419** Reviewed [Csörgő, Sándor](#) Probabilistic approach to limit theorems for the St. Petersburg game. With a preface by Vilmos Totik. *Acta Sci. Math. (Szeged)* 76 (2010), no. 1-2, 233–350. (Reviewer: Anna Jaśkiewicz) [60F05 \(60F99\)](#)



**MR2557893** Reviewed [Csörgő, Sándor](#); [Hatvani, László](#) Stability properties of solutions of linear second order differential equations with random coefficients. *J. Differential Equations* 248 (2010), no. 1, 21–49. (Reviewer: Paulo R. C. Ruffino) [34F15 \(34D20 70L05 93E15\)](#)



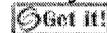
**MR2792541** Reviewed [Csörgő, Sándor](#); [Szabó, Tamás](#) Weighted quantile correlation tests for Gumbel, Weibull and Pareto families. *Probab. Math. Statist.* 29 (2009), no. 2, 227–250. (Reviewer: Gutti J. Babu) [62F05 \(60F05 62E20\)](#)



**MR2656482** Reviewed [Pósfai, Anna](#); [Csörgő, Sándor](#) Asymptotic approximations for coupon collectors. *Studia Sci. Math. Hungar.* 46 (2009), no. 1, 61–96. (Reviewer: Qi Man Shao) [60F05 \(60C05\)](#)



**MR2530112** Reviewed [Kevei, Péter](#); [Csörgő, Sándor](#) Merging of linear combinations to semistable laws. *J. Theoret. Probab.* 22 (2009), no. 3, 772–790. (Reviewer: Makoto Maejima) [60E07 \(60F05\)](#)



**MR2463254** Reviewed [Csörgő, S.](#); [Kevei, P.](#) Merging asymptotic expansions for cooperative gamblers in generalized St. Petersburg games. *Acta Math. Hungar.* 121 (2008), no. 1-2, 119–156. (Reviewer: Krzysztof Szajowski) [60F05 \(60E07 60G40 60G50 91A60\)](#)



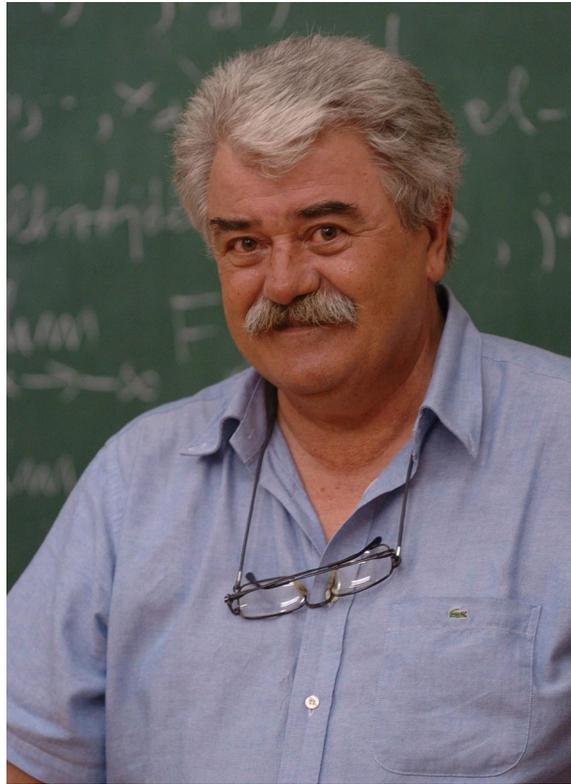
**MR2448396** Reviewed [Csörgő, Sándor](#); [Simons, Gordon](#) Weak laws of large numbers for cooperative gamblers. *Period. Math. Hungar.* 57 (2008), no. 1, 31–60. [65C50](#)



**MR2392789** Reviewed [Csörgő, Sándor](#); [Simons, Gordon](#) St. Petersburg games with the largest gains withheld. *Statist. Probab. Lett.* 77 (2007), no. 12, 1185–1189. [62E15 \(60F15 62G30\)](#)



**MR2339868** Reviewed [Csörgő, Sándor](#) Merging asymptotic expansions in generalized St.





# A glimpse of the KMT (1975) approximation of empirical processes by Brownian bridges via quantiles

MIKLÓS CSÖRGŐ\*

*Dedicated to my brother Sándor, for his sixtieth birthday  
,-Azért vagyunk a világon, hogy valahol otthon legyünk benne.' (Tamási Áron)*

*Communicated by L. Kérchy*

**Abstract.** We deduce a partial version of the KMT (1975) inequality for coupling the uniform empirical process with a sequence of Brownian bridges via the construction used by Csörgő and Révész (CsR) (1978) for their similar coupling of the uniform quantile process with another sequence of Brownian bridges. These constructions are pivoted on the KMT (1975, 1976) inequalities for approximating partial sums by a Wiener process (Brownian motion).

## 1. Introduction and results

Let  $U_1, U_2, \dots$ , be independent uniform  $(0, 1)$  random variables (r.v.'s). For each integer  $n \geq 1$ , define

$$(1.1) \quad \begin{aligned} G_n(t) &:= n^{-1} \sum_{i=1}^n \mathbb{1}\{U_i \leq t\}, \quad 0 \leq t \leq 1, \\ &= \begin{cases} 0, & \text{if } 0 \leq t < U_{1,n}, \\ k/n, & \text{if } U_{k,n} \leq t < U_{k+1,n}, \quad 1 \leq k \leq n-1, \\ 1, & \text{if } U_{n,n} \leq t \leq 1, \end{cases} \end{aligned}$$

---

Received January 8, 2007, and in revised form March 6, 2007.

AMS Subject Classification (2000): 60F17, 60F15, 60G50, 62G30.

\* Supported by an NSERC Canada Discovery Grant at Carleton University, Ottawa.

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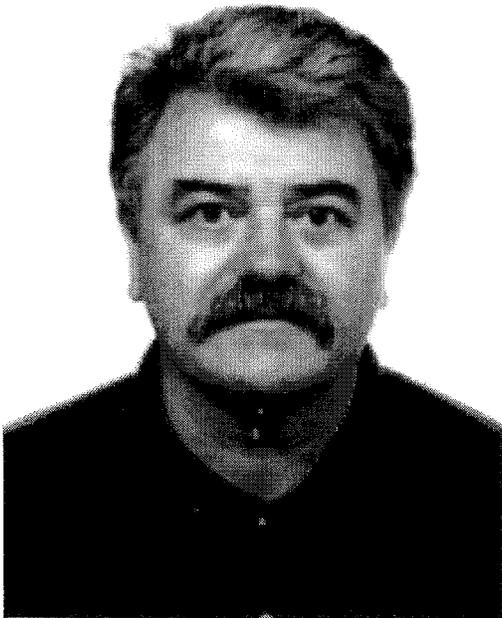




## Magyar Tudomány, 2008/11 1384. o.

### Megemlékezés

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Csörgő Sándor 1947–2008

2008 februárjában tragikus hirtelenséggel elhunyt Csörgő Sándor, az egyik legtermékenyebb és legtöbbet idézett magyar matematikus, a valószínűségszámítás és a matematikai statisztika világviszonylatban kiemelkedő kutatója.

Kis Heves megyei faluban, Egerfarmoson született 1947. július 16-án. A középiskolát Egerben, az egyetemet Szegeden végezte. 1975-ben, Kijevben, Anatolij Vladimirovics Szkorohod vezetésével lett a matematikai tudományok kandidátusa. Ezután a JATE Analízis Alkalmazásai Tanszékén előbb adjunktus, majd docens, végül 1987-től egyetemi (később tanszékvezető egyetemi) tanár. Számos külföldi egyetemen volt vendégprofesszor; 1990-től 1998-ig a University of Michigan (Ann Arbor, Michigan, USA) professzora volt.

Kutatási területe a határeloszlások elmélete és annak alkalmazásai, melyek a klasszikus valószínűségelmélet központi kérdéseikhez tartoznak. Egy tudományos monográfiát tett közzé, és 161 tudományos cikke jelent meg nemzetközi folyóiratokban. Munkáira kétezeröttszáznál több hivatkozás született; ezzel egyike lett azon három leggyakrabban idézett magyar matematikusnak, akik rákerültek a *Science Citation Index* nagy presztízsű listájára (ezen összesen hat magyar tudós található).

Igen erős elméletalkotó képességekkel rendelkezett. Az empirikus karakterisztikus függvények és egyéb transzformáltak valószínűségelméletének megalkotása lényegében az ő nevéhez fűződik, ahogy a legtöbb eddigi statisztikai alkalmazás kezdeményezése vagy kimunkálása is. A megbízhatóságelméleti, illetve orvostudományi alkalmazásoknál fontos szerepet játszó cenzúra alatti empirikus folyamatok approximációs elméletének kiépítését szintén ő kezdte el tanítványaival. Eredendően tisztán elméleti gyökertűek a független, egyforma eloszlású valószínűségi változók összegei határeloszlására vonatkozó – társszerzőkkel folytatott – vizsgálatait, amelyek a valószínűségelmélet egyik központi problémakörébe tartozó,

# OBITUARY: Sándor Csörgő

## 1947–2008

SÁNDOR CSÖRGŐ passed away on February 14, 2008, losing a valiant battle with cancer. He was the Professor in the Department of Stochastics of the Bolyai Institute, University of Szeged, Szeged, Hungary. His death is a tragic loss to the probability and mathematical statistics community.

He was born in Egerfarnos, Hungary on July 16, 1947. He graduated from high school in Eger, and went on to study mathematics at the University of Szeged, where he earned his university diploma. He completed his doctorate under the guidance of Professor Károly Tandori in 1972 with Professor Béla Szőkefalvi-Nagy serving on his examination committee. He obtained his Candidate Degree in 1975 at the Kiev State University under the supervision of Anatoli V. Skorohod, and earned a Doctor of Science Degree in 1984.

Professor Csörgő's scientific career was closely tied to the Bolyai Institute: he became an assistant in 1970, teaching assistant in 1972, Assistant Professor in 1975, Associate Professor in 1978, and Full Professor in 1987. He also held visiting appointments at the University of California, San Diego (1984–85) and the University of North Carolina, Chapel Hill (1989–90). He served as Professor in the Department of Statistics, University of Michigan, Ann Arbor, during the eight academic years in the period 1990–1998.

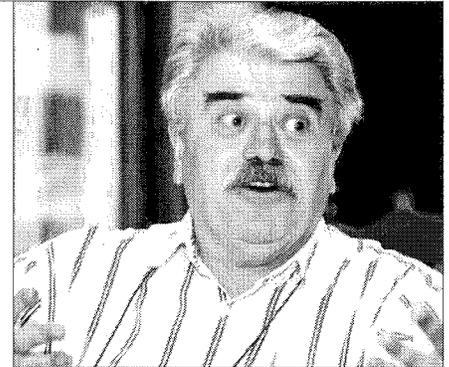
Professor Csörgő's wide-ranging research interests included major areas of probability theory and mathematical statistics. He opened several new fields of research; his contributions to the theory of limit theorems form his most lasting mathematical legacy. He is the coauthor of one research monograph and author of 163 research articles published in international scientific journals.

He was elected IMS Fellow in 1984 and later, Member of the International Statistical Institute. He is one of the three Hungarian mathematicians who appear on the ISI–Highly Cited list of the Science Citation Index. In 2001 he was elected a corresponding member of the Hungarian Academy of Sciences, and in 2007, a full member.

Professor Csörgő founded the Graduate School of Stochastics at the University of Szeged. In fact, he was the first to pursue research in probability theory and mathematical statistics at the Bolyai Institute. Due to his ground-breaking research in this area his school soon won international recognition. One of his duties as head of the Bolyai Institute's Stochastics Program was to design, develop and maintain all of undergraduate and graduate probability and statistics courses at the University of Szeged. He was a dedicated and inspiring teacher and attracted talented students whom he launched into successful scientific careers. Six of his students went on to win prizes at the Hungarian National Scientific Students' Associations Conferences. He supervised four University of Szeged Doctorates, one Candidate Degree and four PhDs, and also advised one Michigan PhD student.

Professor Csörgő was a prominent and active member of the mathematical community. He served on the editorial boards of several international journals, including the *Annals of Statistics* from 1986–88, and regularly refereed research papers and doctoral dissertations. He sat on a number of university and national mathematical education committees, and had served as the Vice President of the Mathematics Section of the Hungarian Academy of Sciences since 2005.

For his distinguished scientific and educational achievements, he was awarded



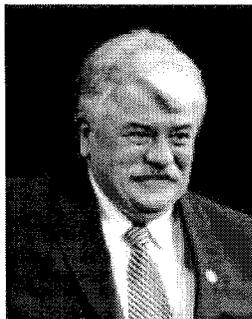
Sándor Csörgő

the 1970 Rényi Kató Memorial Prize, the 1974 Grünwald Géza Memorial Prize, the 1986 Erdős Pál Mathematical Award, the 1999 Award of the Academy, the 2004 Szele Tibor Memorial Prize, the 2005 Master Professor Award of the Hungarian National Conference of Scientific Students' Associations, and the 2005 Szent-Györgyi Albert Prize. In 2007 he was awarded the Grand Prize of the Foundation for Szeged.

On March 15, 2008, Professor Sándor Csörgő posthumously received the prestigious Széchenyi Prize, the highest honour awarded to researchers by the Government of the Republic of Hungary; it is usually presented by the President, the Prime Minister and Speaker of the Hungarian Parliament on the 15th of March national holiday. His widow, Zsuzsi, accepted it in his name.

His untimely death clearly ended a brilliant and highly productive scientific career. His mind was full of research plans until the very end. He continued working with his graduate students even after he became gravely ill. Sadly, his monograph on the St. Petersburg paradox, which he was writing in collaboration with Professor Gordon Simons of the University of North Carolina, Chapel Hill, remains unfinished. His strong and engaging personality, good humor and his unflinching sense of justice and fair play will be sorely missed at the Bolyai Institute as well as in the greater international academic community.

*Bolyai Institute, University of Szeged, and David M. Mason, University of Delaware*



*Professor Sándor Csörgő  
(1947-2008)*

**Sándor Csörgő** passed away on Thursday, February 14<sup>th</sup>, 2008, after a valiant but losing battle with cancer. He was the Professor in the Department of Stochastics of the Bolyai Institute, University of Szeged, Szeged, Hungary. His death is a tragic loss to the probability and mathematical statistics community.

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Professor Csörgő also held one-year visiting appointments at the University of California, San Diego (1984-85) and the University of North Carolina, Chapel Hill (1989-90). He also served as Professor in the Department of Statistics, University of Michigan, Ann Arbor, during the eight academic years in the period 1990-1998.

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several new fields of research; his contributions to the theory of limit theorems form his most lasting mathematical legacy. He is the co-author of one research monograph and author of 163 research articles published in international scientific journals.

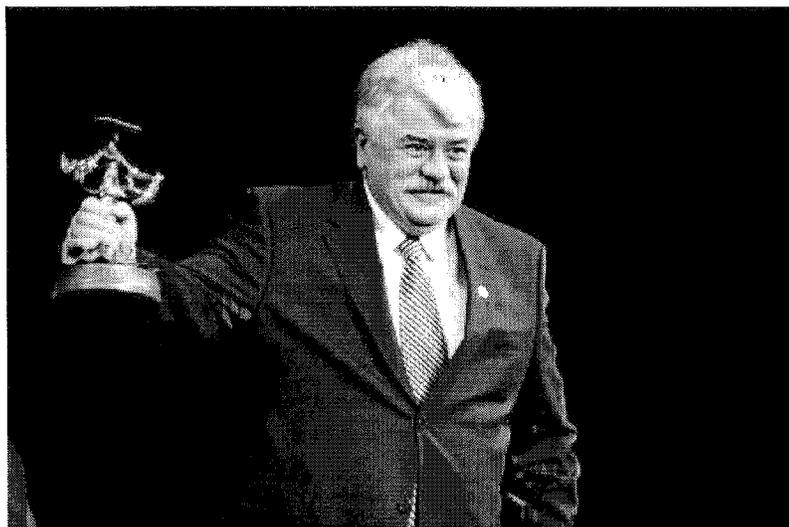
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Professor Csörgő was a prominent and active member of the mathematical community. He served on the editorial boards of several international journals and he contributed regularly his services as a referee for research papers and doctoral dissertations. In particular, he served as Associate Editor for the *Annals of Statistics* from 1986-88 under the Editorship of Willem van Zwet. He sat on a number of important university and national mathematical education committees, and had served as the Vice-President of the Mathematics Section of the Hungarian Academy of Sciences since 2005.

For his distinguished scientific and educational achievements, he was awarded the Rényi Kató Memorial Prize in 1970, the Grünwald Géza Memorial Prize in 1974, the Erdős Pál Mathematical Award in 1986, the Award of the Academy in 1999, the Szele Tibor Memorial Prize in 2004, the Master Professor Award of the Hungarian National Conference of Scientific Students' Associations in 2005, and the

Szent-Györgyi Albert Prize in 2005. In 2007, he was awarded the Grand Prize of the Foundation for Szeged. The photograph below shows him accepting this prize.



On March 15<sup>th</sup>, 2008, Professor Sándor Csörgő posthumously received the prestigious Széchenyi Prize. The Széchenyi Prize is the highest honour awarded to researchers by the Government of the Republic of Hungary; it is usually presented by the President, the Prime Minister and Speaker of the Hungarian Parliament on the 15<sup>th</sup> of March, which is a national holiday. His widow, Zsuzsi, accepted it in his name.

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*Bolyai Institute, University of Szeged*  
*David M. Mason, University of Delaware*



**FEJEZETEK**

**A VALÓSZÍNŰSÉG-  
ELMÉLETBŐL**

**Csörgő Sándor**

