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Szeged Workshop in Convex and Discrete Geometry

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ABSTRACTS

Extremal crosspolytopes and Gaussian vectors

Gergely Ambrus
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Which n-dimensional crosspolytope is extremal with respect to the mean width? Using the classical transformation to Gaussian distributions, the question can be generalised as follows: among the n-dimensional Gaussian random variables X whose covariance matrix has trace 1, which ones maximise and minimise the expectation of $\|X\|_p$ for a fixed p? The geometric question regarding crosspolytopes follows from the $p = \infty$ case. As intuition suggests, the extremal vectors are either two-dimensional or their coordinate variables are i.i.d. Gaussian; however, the roles played by them as minimisers or maximisers depend on n and p. In the talk, we prove the geometric inequality, and investigate the threshold of the problem regarding the Gaussian variables, using the interplay between geometry and probability.

The Cage Problem

Gabriela Araujo-Pardo
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Universidad Nacional Autónoma de México

In this talk we give a brief resume about the Cage Problem and the relationship between the cages of even girth that attain the Moore Bound and the generalized polygons. Moreover, we expose some ideas about our work in this topic and the principal geometric concepts and tools used there.

On minimal tilings with convex cells each containing a unit ball

Károly Bezdek
University of Calgary, Canada, University of Pannonia, and Eötvös University, Hungary

We raise and investigate the following problems that one can regard as very close relatives of the densest sphere packing problem. If the Euclidean 3-space is partitioned into convex cells each containing a unit ball, how should the shapes of the cells be designed to minimize the average surface area (resp., average edge curvature) of the cells? In particular, we prove that the average surface area (resp., average edge curvature) in question is always at least $\frac{24}{\sqrt{3}} = 13.8564...$

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The $T(5)$ property of congruent disks in the plane

TED BISZTRICZKY
University of Calgary, Canada

This is joint work with K. Böröczky and A. Heppes. In the Hadwiger–Debrunner–Klee monograph "Combinatorial geometry in the plane", there is an example of a family of $n > 3$ congruent disks in the plane such that any $n-1$ disks have a transversal (the $T(n-1)$ property) but the n disks do not have a transversal (no $T(n)$ property). The example is due to L. Santaló and the disk centres are the vertices of a regular n-gon.

In the case of $n = 6$ of the example, if the disks have radius 1 then the regular hexagon has edge length $4/3$. We show that this a worst case scenario. Specifically, if a family of $n > 5$ disks of radius 1 is such that the distance between any two disk centres is greater than $4/3$ the $T(5)$ implies $T(n)$.

On the finite set of missing geometric (n_4) point line configurations

JÜRGEN BOKOWSKI
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In the study of combinatorial, topological, or geometric (n_k)-configurations in the projective plane we have n lines, combinatorial ones, pseudolines, or straigth lines, and n points and precisely k of these points are incident with each line and, vice versa, precisely k lines are incident with each point. The AMS research monograph of Grünbaum *Configurations of Points and Lines* from 2009, see [6]. mentions the finite set of unknown (n_4) configurations to be the cases $n = 19, 22, 23, 26, 37, 43$. Oriented matroid techniques, see [1], [2], have been applied to takle these problems, see [3], [4], [7]. The talk will mention algorithms, new constructions, and recent discoveries in this area.

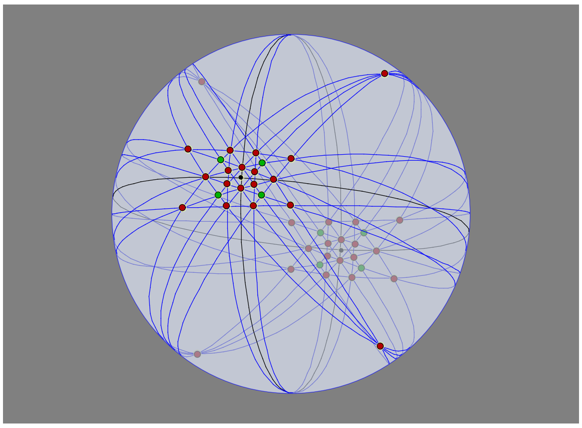

Figure 1: (18_4)-configuration from [4]

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Some families of geometric (n_k) configurations

Gábor Gévay
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In the simplest case, a geometric (n_k) configuration is a set of n points and n lines such that each of the points is incident with precisely k of the lines and each of the lines is incident with precisely k of the points. Instead of lines, the second subset can consist of planes, hyperplanes, circles, or ellipses. Also, the space spanned by such configurations can be either Euclidean or projective space of dimension higher than two. We present some recently discovered classes of configurations of all such types. We also formulate an incidence conjecture concerning a spatial (100_4) point-line configuration.

The normal bundle of a convex body

Peter Gruber
TU Vienna, Austria

We represent the normal bundle of a convex body C in $\mathbb{E}^d$ by a closed convex cone N in $\mathbb{E}^{d^2}$. This cone is studied and several rather unexpected relations between properties of the cone and the convex body are exhibited. In particular, the following topics are considered: Characterization of normal bundle cones. Dimension of N and the ellipsoid character of C. Symmetry. Faces of N and shadow boundaries of C. Lattice packing.

A lattice point inequality for centrally symmetric convex bodies

Matthias Henze
Otto-von-Guericke-University Magdeburg, Germany

In this talk, we present an asymptotically sharp lower bound on the volume in terms of the number of lattice points in centrally symmetric convex bodies. The nonsymmetric analog of this estimate is a classical result of Blichfeldt. Our main tool is a generalization of Davenport's inequality that bounds the number of lattice points in a convex body in terms of volumes of suitable projections.

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Covering the surface of the unit cube by congruent balls

Antal Joós
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The following problem can be read in [1]:
"Let $g(n)$ denote the least number r with the property that the unit square can be covered by n circles of radius r. Determine the exact values of $g(n)$ at least for small integers $n \geq 2$. ... Very little is known about the generalization of the above problem in higher-dimensional spaces."
We generalize this problem in a certain sense:
Let $b(d, n)$ denote the least number r with the property that the surface of the d-dimensional unit cube can be covered by n balls of radius r.
We give the exact value of $b(3, 5)$.

References

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On the k-fold Borsuk numbers of sets

Zsolt Lángi
Budapest University of Technology, Hungary

The problem to find for a bounded set $S \subset \mathbb{R}^n$ the smallest integer k such that S can be written as the union of k sets of diameters strictly smaller than that of S, has been in the focus of scientific research since the 1930s. This problem is called Borsuk's problem, and the number the *Borsuk number* of S. In the past eighty years, many generalizations and variants of this problem have appeared in the literature. In this lecture we propose another one.

We introduce the concept of *k-fold Borsuk numbers* of a bounded set $S \subset \mathbb{R}^n$, and examine their properties. In particular, as time permits, we characterize the k-fold Borsuk numbers of planar sets, give bounds for those of smooth sets and determine them for Euclidean balls. Finally, we examine the k-fold Borsuk numbers of finite point sets in 3-space. As we will see, our generalization can be easily adapted to most variants of Borsuk's problem. Some results are related also to the theory of packings and coverings. The presented topic is a joint work with M. Hujter.

Lattice Points in vector-dilated Polytopes

Eva Linke
Otto-von-Guericke-University Magdeburg, Germany

For $A \in \mathbb{Z}^{m \times n}$ we investigate the behaviour of the number of lattice points in $P_A(b) = \{x \in \mathbb{R}^n : Ax \leq b\}$, depending on the varying vector b. It is known that this number, restricted to a cone of constant combinatorial type of $P_A(b)$, is a quasi-polynomial function if b is an integral vector. We extend this result to rational vectors b and show that the coefficients themselves are piecewise-defined polynomials. To this end, we use a theorem of McMullen on lattice points in Minkowski-sums of rational dilates of rational polytopes and take a closer look at the coefficients appearing there.

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Valuations on Convex Bodies and Sobolev Spaces

Monika Ludwig
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A function Z defined on a lattice $(\mathcal{L}, \vee, \wedge)$ and taking values in an Abelian semigroup is called a *valuation* if

$$Z(f \vee g) + Z(f \wedge g) = Z(f) + Z(g) \tag{1}$$

for all $f, g \in \mathcal{L}$.

A function Z defined on a subset $\mathcal{S}$ of the set $\mathcal{L}$ is called a valuation on $\mathcal{S}$ if

(1) holds whenever $f, g, f \vee g, f \wedge g \in \mathcal{S}$.

The classical case are valuations on convex bodies (compact convex sets) in $\mathbb{R}^n$.

Here valuations are defined on $\mathcal{K}^n$, the space of convex bodies in $\mathbb{R}^n$, which is equipped with the topology coming from the Hausdorff metric. The operations $\vee$ and $\wedge$ are the usual union and intersection.

We give a complete classification of affinely contravariant convex body valued valuations on the Sobolev space $W^{1,1}(\mathbb{R}^n)$. We show that there is a unique such valuation, which turns out to be closely related to the optimal Sobolev body introduced by Lutwak, Yang & Zhang. The result is based on a classification of convex body valued valuations on $\mathcal{K}^n$.

Ball characterizations (joint results with J. Jerónimo-Castro)

E. Makai, Jr.
MTA Rényi Institute, Hungary

R. High proved the following theorem. If the intersections of any two congruent copies of a plane convex body are centrally symmetric, then the body is a circle. We prove several generalizations of this theorem.

Let X be a space of constant curvature, i.e., S^d, $\mathbb{R}^d$ or H^d, where $d \geq 2$. Let $K, L \subset X$ be closed convex sets with non-empty interiors, such that the intersections $(\varphi K) \cap (\psi L)$ of any two congruent copies of them are centrally symmetric. Then, under a regularity assumption $(C2_+)$, K and L are congruent balls.

For the 2-dimensional case we have more exact results. Under some rather mild hypotheses, we can describe all those pairs $K, L \subset X$ of closed convex sets with interior points, such that the intersections $(\varphi K) \cap (\psi L)$ of any congruent copies of them have some non-trivial symmetry.

For $X = \mathbb{R}^d$, V. Soltan proved that if the intersections $(K + x) \cap (L + y)$ of any two translates of the convex bodies $K, L \subset \mathbb{R}^d$ are centrally symmetric, then K and L are mirror images of each other w.r.t. some point. For $X = \mathbb{R}^d$, we prove the analogous statement, for $\operatorname{conv}[(K + x) \cup (L + y)]$, rather than $(K + x) \cap (L + y)$. Without any additional hypotheses, we can describe all pairs $K, L \subset \mathbb{R}^d$ of closed convex sets with interior points, such that the intersections/closed convex hulls of the unions $(\varphi K) \cap (\psi L)/\operatorname{conv}[(\varphi K) \cup (\psi L)]$ of any of their congruent copies are centrally symmetric.

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Topological Berge and Breen's Theorems

Luis Montejano
UNAM, México

Some strange results about transversals to families of convex sets are achieved by means of two topological versions of Berge and Breen's Theorems.

Push Forward Measures and Concentration Phenomena (joint work with C. Hugo Jiménez and Rafael Villa)

Márton Naszódi
Eötvös University, Hungary

Consider a centrally symmetric convex body K endowed with a measure μ, and another convex body L. We study how well concentration properties of μ are inherited by the push-forward measure $\pi_*(\mu)$ on L, where $\pi : K \to L$ denotes the $x \mapsto \frac{x}{\|x\|_L}\|x\|_K$ central projection. We found that concentration is well transported between certain pairs of bodies that are far apart in the Banach–Mazur sense. We consider also the question of how far the cube is from being equipable by a measure of good concentration.

About piercing numbers of affine planes, lines and intervals

Deborah Oliveros
Instituto de Matemáticas, UNAM, México

In this talk, we will present an interesting family of r-hypergraphs with the property, that the chromatic number is bounded from above by a function of its clique number. Bounds that allows us to find the piercing numbers of some families of affine hyperplanes, lines and intervals.

Bonnesen-style inradius inequalities

E. Saorín Gómez
Otto-von-Guericke Universität Magdeburg, Germany

Let $E \subset \mathbb{R}^n$ be a convex body with interior points and B_n the n-dimensional unit ball. The Bonnesen–Blaschke inequality for a planar convex body K establishes that

$$\mathrm{W}_1(K;E)^2 - \mathrm{V}(K)\mathrm{V}(E) \geq \frac{\mathrm{V}(E)^2}{4}(\mathrm{R}(K;E) - \mathrm{r}(K;E))^2 \tag{2}$$

where $\mathrm{W}_1(K;E)$ is the first quermassintegral of K w.r.t. E and
$mathrmr(K;E)$ and $\mathrm{R}(K;E)$ are the inradius and the circumradius of K w.r.t. E.

An extension of Bonnesen's inradius inequality to higher dimensions was conjectured by Wills and proved simultaneously by Bokowski and Diskant for $E = B_n$:

$$\mathrm{V}(K) - n\mathrm{r}(K;B_n)\mathrm{W}_1(K;B_n) + (n-1)\mathrm{r}(K;B_n)^n\mathrm{V}(B_n) \leq 0. \tag{3}$$

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Sangwine-Yager proved it for a general relative body E with interior points, as a consequence of a much more general result which bounded the volume of every inner parallel body of K in terms of the quermassintegrals of K and some mixed volumes involving inner parallel bodies.

We provide new inequalities for the volume of (the inner parallel bodies of) a convex body in terms of the quermassintegrals of it, using the technique of inner parallel bodies. These bounds are obtained as consequences of, on the one hand, inequalities for inner parallel bodies involving mixed volumes and, on the other hand, inequalities which relate a convex body with its inner parallel bodies, its kernel and its form body.

Diametric completions

Rolf Schneider
University of Freiburg, Germany

A nonempty bounded subset M of a metric space is called *diametrically complete* if any subset of the space strictly containing M has larger diameter than M. In a Euclidean space, the diametrically complete sets are precisely the convex bodies of constant width. In a Minkowski space (a finite-dimensional real normed space) of dimension greater than two, there are in general few bodies of constant width, but many diametrically complete sets. Every bounded set is contained in a diametrically complete set of the same diameter (necessarily a convex body, and far from unique, in general), called a *completion* of the given set. We report on results about the following topics in Minkowski spaces: comparison of constant width and diametric completeness, the set of all diametrically complete sets, the set of completions of a given set, Lipschitz continuous selections of completions. (This is joint work with José Pedro Moreno).

Semi-inner product und its application in the geometry of normed spaces

Margarita Spirova
TU Chemnitz, Germany

The semi-inner product in Banach spaces was defined by Lumer in [Semi-inner-product spaces, *Trans. Amer. Math. Soc.* **123** (1967), 436-446]. In this way he carried over Hilbert-space arguments to the theory of Banach spaces. We consider finite dimensional real Banach (or normed) spaces and present some geometric aspects of semi-inner product. We also discuss how the semi-inner product structure of a normed space $(\mathbb{B}, \|\cdot\|)$ does relate to the dual space of $\mathbb{B}$ and the anti-normed space of $(\mathbb{B}, \|\cdot\|)$.

A Schütte theorem for the 4-norm

Konrad Swanepoel
Londons School of Economics, U.K.

The well-known theorem of Schütte gives a sharp lower bound for the ratio of the maximum distance and minimum distance between $d+2$ points in d-dimensional Euclidean space. We discuss an analogue for the space ℓ_4^d, where the norm is given by $\|(x_1, x_2, \ldots, x_d)\|_4 = \left(\sum_{i=1}^d x_i^4\right)^{1/4}$. This gives a new proof that the maximum number of points in an equilateral set in ℓ_4^d is $d+1$.

The proof is analogous to Bárány's proof of the classical Schütte theorem.

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On the difference between the Hadwiger number and the lattice kissing number of a convex body

István Talata
Ybl Faculty of Szent István University, Hungary

The Hadwiger number $H(K)$ of a d-dimensional convex body K is the maximum number of neighbours that a body can have in a packing with translates of K. (In a packing, two convex bodies are called neighbours if they touch each other, that is, they have a non-empty intersection.) The lattice kissing number $H_L(K)$ is defined analogously, with the further restriction that the translation vectors corresponding to the translates of K in the packing form a lattice in $\mathbb{R}^d$. It is known that $H(K) \leq 3^d - 1$ (Hadwiger, 1957). Furthermore, there is a d-dimensional convex body K_d for every $d \geq 4$ such that $H(K_d) - H_L(K_d) \geq (\sqrt{7})^{d-o(d)}$ (Talata, 2005). We now improve on this lower bound to show that there exists a d-dimensional convex body K_d for every $d \geq 4$ such that $H(K_d) - H_L(K_d) \geq c \cdot 3^d$ for some absolute constant $c > 0$.

Siegel's Lemma with restrictions

Carsten Thiel
Otto-von-Guericke-Universität, Magdeburg, Germany

The classical Siegel's Lemma asks for a small non-zero integral solution to a system of linear equations with integer coefficients. In recent work by Fukshansky additional restrictions have been imposed, forbidding the solution to be contained in a collection of sublattices.

In this talk, which is based on joint work with Martin Henk, we generalise the geometric idea behind Fukshansky's results: Given a convex body K, a lattice Λ and a collection $\Lambda_1, \ldots, \Lambda_m \subset \Lambda$ of proper sublattices, what is the minimal γ such that γK contains a point $x \in \Lambda \setminus \bigcup_i \Lambda_i$?

The Equivalence of the Illumination and Covering Conjectures

Ryan Trelford
University of Calgary, Canada

Let K be a convex body in E^d, and let v be any non-zero vector (referred to as a direction). A point P on the boundary of K is said to be illuminated by v if the ray emanating from P with direction v intersects the interior of K. One can ask what is the smallest positive integer n such that there exists a set of distinct directions $\{v_1, ..., v_n\}$ whereby every boundary point of K is illuminated by at least one of the v_i's. The illumination conjecture (formulated by I. Gohberg and A. Markus) states that n is at most 2^d. Surprisingly, 2^d is also the conjectured maximum number of smaller homothetic copies of K that are required to cover K (conjectured by H. Hadwiger and V. Boltyanski). In this talk, I will outline the proof that the Illumination Conjecture and the Covering Conjecture are indeed equivalent.

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Simplicial convexity

Tudor Zamfirescu
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By Carathodory's theorem, a convex body in Euclidean d-space can be produced as the union of all d-dimensional simplices with vertices in some small set. This can also be done using simplices of smaller dimension, if we iterate the procedure. This kind of generation of convex bodies was studied half a century ago by Bonnice and Klee. Calling the result at any stage *simplicially convex*, we get an interesting generalization of convexity, some properties of which shall be discussed in this talk.

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