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The existence of positive solutions for nonlinear boundary system with p-Laplacian operator based on sign-changing nonlinearities*

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Abstract

In this paper, we study a nonlinear boundary value system with p-Laplacian operator

$$\begin{cases} (\phi_{p_1}(u'))' + a_1(t)f(u,v) = 0, \quad 0 < t < 1, \\ (\phi_{p_2}(v'))' + a_2(t)g(u,v) = 0, \quad 0 < t < 1, \\ \alpha_1\phi_{p_1}(u(0)) - \beta_1\phi_{p_1}(u'(0)) = \gamma_1\phi_{p_1}(u(1)) + \delta_1\phi_{p_1}(u'(1)) = 0, \\ \alpha_2\phi_{p_2}(v(0)) - \beta_2\phi_{p_2}(v'(0)) = \gamma_2\phi_{p_2}(v(1)) + \delta_2\phi_{p_2}(v'(1)) = 0, \end{cases}$$

where $\phi_{p_i}(s) = |s|^{p_i-2}s,\ p_i > 1, i = 1, 2.$ We obtain some sufficient conditions for the existence of two positive solutions or infinitely many positive solutions by using a fixed-point theorem in cones. Especially, the nonlinear terms f, g are allowed to change sign. The conclusions essentially extend and improve the known results.

Key words: p-Laplacian operator; nonlinear boundary value problems; positive solutions.

1 Introduction

In this paper, we study the existence of positive solutions for nonlinear singular boundary value system with p-Laplacian operator

$$\begin{cases} (\phi_{p_1}(u'))' + a_1(t)f(u,v) = 0, \quad 0 < t < 1, \\ (\phi_{p_2}(v'))' + a_2(t)g(u,v) = 0, \quad 0 < t < 1, \\ \alpha_1\phi_{p_1}(u(0)) - \beta_1\phi_{p_1}(u'(0)) = \gamma_1\phi_{p_1}(u(1)) + \delta_1\phi_{p_1}(u'(1)) = 0, \\ \alpha_2\phi_{p_2}(v(0)) - \beta_2\phi_{p_2}(v'(0)) = \gamma_2\phi_{p_2}(v(1)) + \delta_2\phi_{p_2}(v'(1)) = 0, \end{cases} \tag{1.1}$$

where $\phi_{p_i}(s)$ are p-Laplacian operator; i.e., $\phi_{p_i}(s) = |s|^{p_i-2}s,\ p_i > 1$, and $a_i(t) : (0,1) \to [0, +\infty)$, $\phi_{q_i} = (\phi_{p_i})^{-1}, \dfrac{1}{p_i} + \dfrac{1}{q_i} = 1,\ \alpha_i > 0,\ \beta_i \geq 0,\ \gamma_i > 0,\ \delta_i \geq 0,\ i = 1, 2.$

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In recent years, because of the wide mathematical and physical background [1, 15], the existence of positive solutions for nonlinear boundary value problems with p-Laplacian operator received wide attention. Especially, when $p = 2$ or $\phi_p(u) = u$ is linear, the existence of positive solutions for nonlinear singular boundary value problems has been obtained (see [6, 10, 12, 16]); when $p \neq 2$ or $\phi_p(u) \neq u$ is nonlinear, papers [7, 11, 13, 14, 17] have obtained many results by using comparison results or topological degree theory.

In [10], Kaufmann and Kosmatov established the existence of countably many positive solutions for the following two-point boundary value problem

$$\begin{cases} u''(t) + a(t)f(u(t)) = 0, \ 0 < t < 1, \\ u'(0) = 0, \ u(1) = 0, \end{cases} \tag{1.2}$$

where $a \in L^p[0, 1]$, $p \geq 1$, and $a(t)$ has countably singularities on $[0, \frac{1}{2})$.

Very recently, authors [13] studied the boundary value problem

$$\begin{cases} (\phi_p(u'))' + a(t)f(u) = 0, \quad 0 < t < 1, \\ \alpha\phi_p(u(0)) - \beta\phi_p(u'(0)) = 0, \ \gamma\phi_p(u(1)) + \delta\phi_p(u'(1)) = 0, \end{cases} \tag{1.3}$$

where $\phi_p(s)$ is p-Laplacian operator; i.e., $\phi_p(s) = |s|^{p-2}s$, $p > 1$, and $a(t) : (0, 1) \to [0, +\infty)$, $\phi_q = (\phi_p)^{-1}, \dfrac{1}{p} + \dfrac{1}{q} = 1$, $\alpha > 0$, $\beta \geq 0$, $\gamma > 0$, $\delta \geq 0$. Using a fixed-point theorem, we obtained the existence of positive solutions or infinitely many positive solutions for boundary value problems (1.3).

In [14], authors studied the boundary value system (1.1) by applying the fixed-point theorem of cone expansion and compression of norm type. We obtained the existence of infinitely many positive solutions for problems (1.1).

It is well known that the key condition used in the above papers is that the nonlinearity is nonnegative. If the nonlinearity is negative somewhere, then the solution needs no longer be concave down. As a result it is difficult to find positive solutions of the p-Laplacian equation when f changes sign.

In 2003, Agarwal, Lü and O'Regan [2] investigated the singular boundary value problem

$$\begin{cases} (\phi_p(y'))' + q(t)f(t, y(t)) = 0, \quad t \in (0, 1), \\ y(0) = y(1) = 0, \end{cases} \tag{1.4}$$

by means of the upper and lower solution method, where the nonlinearity f is allowed to change sign.

In [8], Ji, Feng and Ge studied the existence of multiple positive solutions for the following boundary value problem

$$\begin{cases} (\phi_p(u'))' + a(t)f(t, u(t)) = 0, \quad t \in (0, 1), \\ u(0) = \sum\limits_{i=1}^{m} a_i u(\xi_i), \quad u(1) = \sum\limits_{i=1}^{m} b_i u(\xi_i), \end{cases} \tag{1.5}$$

where $0 < \xi_1 < \cdots < \xi_m < 1$, a_i, $b_i \in [0, +\infty)$ satisfy $0 < \sum\limits_{i=1}^{m-2} a_i, \sum\limits_{i=1}^{m-2} b_i < 1$. The nonlinearity f is allowed to change sign.

In [9], Ji, Tian and Ge researched the existence of positive solutions for the boundary value problem

$$\begin{cases} (\phi_p(u'))' + f(t,u,u') = 0, & t \in [0,1], \\ u'(0) = \sum\limits_{i=1}^{m} a_i u'(\xi_i), & u(1) = \sum\limits_{i=1}^{m} b_i u(\xi_i). \end{cases} \tag{1.6}$$

They showed that problem (1.6) has at least one or two positive solutions under some assumptions by applying a fixed point theorem. The interesting points are that the nonlinear term f is involved with the first-order derivative explicitly and f may change sign.

To date no paper has appeared in the literature which discusses the coupled systems with one-dimensional p-Laplacian when nonlinearity in the differential equations may change sign. This paper attempts to fill this gap in the literature.

In the rest of the paper, we make the following assumptions:

(H_1) $f, g \in C([0,+\infty) \times [0,+\infty), (-\infty,+\infty))$, $\alpha_i > 0,\ \beta_i \geq 0,\ \gamma_i > 0,\ \delta_i \geq 0,\ (i = 1,2)$;

(H_2) $a_i \in C[(0,1),[0,\infty)]$ and

$$0 < \int_0^1 a_i(t)dt < \infty, 0 < \int_0^1 \phi_{q_i}(\int_0^s a_i(r)dr)ds < \infty,\ i = 1,2;$$

(H_3) $f(0,v) \geq 0, g(u,0) \geq 0$, for $t \in (0,1)$ and $a_1(t)f(0,v), a_2(t)g(u,0)$ are not identically zero on any subinterval of $(0,1)$.

2 Preliminaries and Lemmas

In this section, we give some preliminaries and definitions.

Definition 2.1. Let E be a real Banach space over R . A nonempty closed set $P \subset E$ is said to be a cone provided that

(i) $au \in P$ for all $u \in P$ and all $a \geq 0$ and

(ii) $u,\ -u \in P$ implies $u = 0$.

The following well-known result of the fixed point index is crucial in our arguments.

Theorem 2.1.[See 3-5] Let X be a real Banach space and K be a cone subset of X. Assume $r > 0$ and that $T : \overline{K_r} \longrightarrow X$ be a completely continuous operator such that $Tx \neq x$ for $x \in \partial K_r = \{x \in K : ||x|| = r\}$. Then the following assertions hold:

(i) If $||Tx|| \geq ||x||$, for $x \in \partial K_r$, then $i(T, K_r, K) = 0$.

(ii) If $||Tx|| \leq ||x||$, for $x \in \partial K_r$, then $i(T, K_r, K) = 1$.

Let $E = C[0,1] \times C[0,1]$, then E is a Banach space with the norm $\|(u,v)\| = \|u\| + \|v\|$, where $\|u\| = \sup\limits_{t\in[0,1]} |u(t)|,\ \|v\| = \sup\limits_{t\in[0,1]} |v(t)|$. For $(x,y),\ (u,v) \in E$, we note that $(x,y) \leq (u,v) \Leftrightarrow x \leq u,\ y \leq v$. Let

$$K = \{(u,v) \in E : u(t) \geq 0,\ v(t) \geq 0\}.$$

$$K' = \{(u,v) \in E : u(t) \geq 0,\ v(t) \geq 0, u(t),\ v(t) \text{ are concave on [0,1]}\}.$$

Then K, K' are cones of E.

Let $K_r = \{(u,v) \in K, ||(u,v)|| < r\}$, then $\partial K_r = \{(u,v) \in K, ||(u,v)|| = r\}$, $\overline{K_r} = \{(u,v) \in K, ||(u,v)|| \leq r\}$, $u^+(t) = \max\{u(t), 0\}$.

Lemma 2.1.[See 13-14] Suppose that condition (H_2) holds, then there exists a constant $\eta \in (0, \frac{1}{2})$ which satisfies $0 < \int_\eta^{1-\eta} a_i(t)dt < \infty, i = 1, 2$. Furthermore, the functions

$$A_i(t) = \int_\eta^t \phi_{q_i}\left(\int_s^t a_i(r)dr\right) ds + \int_t^{1-\eta} \phi_{q_i}\left(\int_t^s a_i(r)dr\right) ds,\ t \in [\eta, 1-\eta], i = 1, 2$$

are positive and continuous on $[\eta, 1-\eta]$, and therefore $A_i(t) (i = 1, 2)$ have minimums on $[\eta, 1-\eta]$. Hence we suppose that there exists $L > 0$ such that $A_i(t) \geq L,\ t \in [\eta, 1-\eta], \quad i = 1, 2$.

Lemma 2.2. Let $X = C[0, 1], P = \{u \in X : u \geq 0\}$. Suppose $T : X \to X$ is completely continuous. Define $\theta : TX \to P$ by

$$(\theta y)(t) = \max\{y(t), 0\}, \quad \text{for } \ y \in TX.$$

Then

$$\theta \circ T : P \to P$$

is also a completely continuous operator.

Proof. The complete continuity of T implies that T is continuous and maps each bounded subset in X to a relatively compact set. Denote θy by $\overline{y}$.

Given a function $h \in C[0, 1]$, for each $\varepsilon > 0$ there exists $\delta > 0$ such that

$$||Th - Tg|| < \varepsilon, \quad \text{for } \ g \in X, ||g - h|| < \delta.$$

Since

$$\begin{aligned} |(\theta Th)(t) - (\theta Tg)(t)| &= |\max\{(Th)(t), 0\} - \max\{(Tg)(t), 0\}| \\ &\leq |(Th)(t) - (Tg)(t)| < \varepsilon, \end{aligned}$$

we have

$$||(\theta T)h - (\theta T)g|| < \varepsilon, \quad \text{for } \ g \in X, ||g - h|| < \delta,$$

and so θT is continuous.

For any arbitrary bounded set $D \subset X$ and $\forall \varepsilon > 0$, there are $y_i (i = 1, 2, \cdots, m)$ such that

$$TD \subset \bigcup_{i=1}^m B(y_i, \varepsilon),$$

where $B(y_i, \varepsilon) = \{u \in X : ||u - y_i|| < \varepsilon\}$. Then, for $\forall \overline{y} \in (\theta \circ T)D$, there is a $y \in TD$ such that $\overline{y}(t) = \max\{y(t), 0\}$. We choose $i \in \{1, 2, \cdots, m\}$ such that $||y - y_i|| < \varepsilon$. The fact

$$\max_{t\in[0,1]} |\overline{y}(t) - \overline{y}_i(t)| \leq \max_{t\in[0,1]} |y(t) - y_i(t)|,$$

which implies $\overline{y} \in B(\overline{y}_i, \varepsilon)$. Hence $(\theta \circ T)D$ has a finite $\varepsilon-$net and $(\theta \circ T)D$ is relatively compact.

Lemma 2.3.[See 11] Let $(u, v) \in K'$ and η of Lemma 2.1, then

$$u(t) + v(t) \geq \eta \|(u, v)\|, t \in [\eta, 1-\eta].$$

Now we consider the boundary value system (1). Firstly, we define a mapping $A : K \to E$:

$$A(u, v)(t) = (A_1(u, v), A_2(u, v))(t),$$

given by

$$A_1(u,v)(t) = \begin{cases} \phi_{q_1}\left(\frac{\beta_1}{\alpha_1}\int_0^{\sigma_{1(u,v)}} a_1(r)f(u(r),v(r))dr\right) \\ +\int_0^t \phi_{q_1}\left(\int_s^{\sigma_{1(u,v)}} a_1(r)f(u(r),v(r))dr\right)ds, & 0 \le t \le \sigma_{1(u,v)}, \\ \phi_{q_1}\left(\frac{\delta_1}{\gamma_1}\int_{\sigma_{1(u,v)}}^1 a_1(r)f(u(r),v(r))dr\right) \\ +\int_t^1 \phi_{q_1}\left(\int_{\sigma_{1(u,v)}}^s a_1(r)f(u(r),v(r))dr\right)ds, & \sigma_{1(u,v)} \le t \le 1. \end{cases}$$

$$A_2(u,v)(t) = \begin{cases} \phi_{q_2}\left(\frac{\beta_2}{\alpha_2}\int_0^{\sigma_{2(u,v)}} a_2(r)g(u(r),v(r))dr\right) \\ +\int_0^t \phi_{q_2}\left(\int_s^{\sigma_{2(u,v)}} a_2(r)g(u(r),v(r))dr\right)ds, & 0 \le t \le \sigma_{2(u,v)}, \\ \phi_{q_2}\left(\frac{\delta_2}{\gamma_2}\int_{\sigma_{2(u,v)}}^1 a_2(r)g(u(r),v(r))dr\right) \\ +\int_t^1 \phi_{q_2}\left(\int_{\sigma_{2(u,v)}}^s a_2(r)g(u(r),v(r))dr\right)ds, & \sigma_{2(u,v)} \le t \le 1. \end{cases}$$

It is clear that the existence of a positive solution for the boundary value system (1.1) is equivalent to the existence of a nontrivial fixed point of A in K (see for example [14]).

Next, for any $(u,v) \in K$, define

$$B(u,v)(t) = (B_1(u,v)(t), B_2(u,v)(t)),$$

where

$$B_1(u,v)(t) = \begin{cases} \Big[\phi_{q_1}\left(\frac{\beta_1}{\alpha_1}\int_0^{\sigma_{1(u,v)}} a_1(r)f(u(r),v(r))dr\right) \\ +\int_0^t \phi_{q_1}\left(\int_s^{\sigma_{1(u,v)}} a_1(r)f(u(r),v(r))dr\right)ds\Big]^+, & 0 \le t \le \sigma_{1(u,v)}, \\ \Big[\phi_{q_1}\left(\frac{\delta_1}{\gamma_1}\int_{\sigma_{1(u,v)}}^1 a_1(r)f(u(r),v(r))dr\right) \\ +\int_t^1 \phi_{q_1}\left(\int_{\sigma_{1(u,v)}}^s a_1(r)f(u(r),v(r))dr\right)ds\Big]^+, & \sigma_{1(u,v)} \le t \le 1. \end{cases}$$

$$B_2(u,v)(t)=\begin{cases}\Big[\phi_{q_2}\left(\dfrac{\beta_2}{\alpha_2}\displaystyle\int_0^{\sigma_{2(u,v)}} a_2(r)g(u(r),v(r))dr\right)\\ +\displaystyle\int_0^t \phi_{q_2}\left(\int_s^{\sigma_{2(u,v)}} a_2(r)g(u(r),v(r))dr\right)ds\Big]^+, & 0\le t\le \sigma_{2(u,v)},\\ \Big[\phi_{q_2}\left(\dfrac{\delta_2}{\gamma_2}\displaystyle\int_{\sigma_{2(u,v)}}^1 a_2(r)g(u(r),v(r))dr\right)\\ +\displaystyle\int_t^1 \phi_{q_2}\left(\int_{\sigma_{2(u,v)}}^s a_2(r)g(u(r),v(r))dr\right)ds\Big]^+, & \sigma_{2(u,v)}\le t\le 1.\end{cases}$$

For $(u,v)\in E$, define $T:E\to K$ by $T(u,v)=(u^+,v^+)$. By Lemma 2.2, we have $B=TA$.

Finally, for any $(u,v)\in K'$, define

$$F(u,v)(t)=(F_1(u,v)(t),F_2(u,v)(t)),$$

given by

$$F_1(u,v)(t)=\begin{cases}\phi_{q_1}\left(\dfrac{\beta_1}{\alpha_1}\displaystyle\int_0^{\sigma_{1(u,v)}} a_1(r)f^+(u(r),v(r))dr\right)\\ +\displaystyle\int_0^t \phi_{q_1}\left(\int_s^{\sigma_{1(u,v)}} a_1(r)f^+(u(r),v(r))dr\right)ds, & 0\le t\le \sigma_{1(u,v)},\\ \phi_{q_1}\left(\dfrac{\delta_1}{\gamma_1}\displaystyle\int_{\sigma_{1(u,v)}}^1 a_1(r)f^+(u(r),v(r))dr\right)\\ +\displaystyle\int_t^1 \phi_{q_1}\left(\int_{\sigma_{1(u,v)}}^s a_1(r)f^+(u(r),v(r))dr\right)ds, & \sigma_{1(u,v)}\le t\le 1.\end{cases}$$

$$F_2(u,v)(t)=\begin{cases}\phi_{q_2}\left(\dfrac{\beta_2}{\alpha_2}\displaystyle\int_0^{\sigma_{2(u,v)}} a_2(r)g^+(u(r),v(r))dr\right)\\ +\displaystyle\int_0^t \phi_{q_2}\left(\int_s^{\sigma_{2(u,v)}} a_2(r)g^+(u(r),v(r))dr\right)ds, & 0\le t\le \sigma_{2(u,v)},\\ \phi_{q_2}\left(\dfrac{\delta_2}{\gamma_2}\displaystyle\int_{\sigma_{2(u,v)}}^1 a_2(r)g^+(u(r),v(r))dr\right)\\ +\displaystyle\int_t^1 \phi_{q_2}\left(\int_{\sigma_{2(u,v)}}^s a_2(r)g^+(u(r),v(r))dr\right)ds, & \sigma_{2(u,v)}\le t\le 1.\end{cases}$$

With respect to operator $F_1(u,v)$, because of

$$(F_1(u,v))'(t)=\begin{cases}\phi_{q_1}\left(\displaystyle\int_t^{\sigma_{1(u,v)}} a_1(r)f^+(u(r),v(r))dr\right)\ge 0, & 0\le t\le \sigma_{1(u,v)},\\ -\phi_{q_1}\left(\displaystyle\int_{\sigma_{1(u,v)}}^t a_1(r)f^+(u(r),v(r))dr\right)\le 0, & \sigma_{1(u,v)}\le t\le 1.\end{cases}$$

So the operator F_1 is continuous and $F_1(u,v)'(\sigma_{1(u,v)})=0$, and for any $(u,v)\in K'$, we have

$$(\phi_{q_1}(F_1(u,v)')(t))'=-a_1(t)f^+(u(t),v(t)),\quad a.e.\ t\in(0,1),$$

and $F_1(u,v)(\sigma_{1(u,v)}) = \|F_1(u,v)\|$. Therefore we have $F_1(u,v)(t)$ is concave function. Similarly, we have $F_2(u,v)(t)$ is also concave function. Thus $F(K') \subset K'$, and $||F(u,v)|| = F_1(u,v)(\sigma_{1(u,v)}) + F_2(u,v)(\sigma_{2(u,v)})$.

3 The existence of two positive solutions

For convenience, we set

$$M_i = 2\left[1 + \phi_{q_i}(\frac{\beta_i}{\alpha_i})\right]\phi_{q_i}(\int_0^1 a_i(r)dr), \quad 0 < N_i < \frac{L}{2},\ i = 1,2.$$

In this section, we will discuss the existence of two positive solutions.

Theorem 3.1. Suppose that conditions (H_1), (H_2) and (H_3) hold. And assume that there exist positive numbers $a,\ b,\ d$ such that $0 < \frac{d}{\eta} < a < \eta b < b$ and f, g satisfy the following conditions

(H_4): $\quad f(u,v) \geq 0,\ \ g(u,v) \geq 0, \quad$ for $u + v \in [d,b]$;

(H_5): $\quad f(u,v) < \phi_{p_1}(\dfrac{a}{M_1}),\ g(u,v) < \phi_{p_2}(\dfrac{a}{M_2}), \quad$ for $u+v \in [0,a]$;

(H_6): $\quad f(u,v) > \phi_{p_1}(\dfrac{b}{N_1}),\ g(u,v) > \phi_{p_1}(\dfrac{b}{N_2}), \quad$ for $u+v \in [\eta b, b]$.

Then, the boundary value system (1.1) has at least two positive solutions (u_1,v_1) and (u_2,v_2) such that

$$0 \leq ||(u_1,v_1)|| < a < ||(u_2,v_2)|| < b.$$

Proof. First of all, from the definitions of B and F, it is clear that $B(K) \subset K$ and $F(K') \subset K'$. Moreover, by (H_2) and the continuity of f, g, it is easy to see that $A : K \to X$ and $F : K' \to K'$ are completely continuous. Using Lemma 2.2, we have $B = TA : K \to K$ and B is completely continuous.

Now we prove that B has a fixed point $(u_1,v_1) \in K$ with $0 < ||(u_1,v_1)|| < a$. In fact, $\forall (u,v) \in \partial K_a$, then $||(u,v)|| = a$ and $0 < u(t)+v(t) \leq a$, from (H_5) we have

$$\begin{aligned}
||B_1(u,v)|| \quad &= \max\nolimits_{t\in[0,1]}\Bigg[\phi_{q_1}\left(\frac{\beta_1}{\alpha_1}\int_0^{\sigma_{1(u,v)}} a_1(r)f(u(r),v(r))dr\right) \\
&+ \int_0^t \phi_{q_1}\left(\int_s^{\sigma_{1(u,v)}} a_1(r)f(u(r),v(r))dr\right)ds\Bigg]^+ \\
&\leq \max\nolimits_{t\in[0,1]}\max\Bigg\{\phi_{q_1}\left(\frac{\beta_1}{\alpha_1}\int_0^{\sigma_{1(u,v)}} a_1(r)f(u(r),v(r))dr\right) \\
&+ \int_0^t \phi_{q_1}\left(\int_s^{\sigma_{1(u,v)}} a_1(r)f(u(r),v(r))dr\right)ds, 0\Bigg\} \\
&< \frac{a}{M_1}\left[1+\phi_{q_1}(\frac{\beta_1}{\alpha_1})\right]\left(\phi_{q_1}(\int_0^1 a_1(r)dr)ds\right) = \frac{a}{2}.
\end{aligned}$$

Similarly, we get

$$||B_2(u,v)|| < \frac{a}{2}.$$

Thus,

$$||B(u,v)|| = ||B_1(u,v)|| + ||B_2(u,v)|| < \frac{a}{2} + \frac{a}{2} = a = ||(u,v)||.$$

It follows from Theorem 2.1 that

$$i(B, K_a, K) = 1,$$

and hence B has a fixed point $(u_1, v_1) \in K$ with $0 < ||(u_1, v_1)|| \leq a$. Obviously, (u_1, v_1) is a solution of boundary value system(1.1) if and only if (u_1, v_1) is a fixed point of A.

Next, we need to prove that (u_1, v_1) is a fixed point of A. If not, then $A(u_1, v_1) \neq (u_1, v_1)$, i.e., $A_1(u_1, v_1) \neq u_1$ or $A_2(u_1, v_1) \neq v_1$. Without loss generality, suppose $A_1(u_1, v_1) \neq u_1$, then there exists $t_0 \in (0, 1)$ such that $u_1(t_0) \neq A_1(u_1, v_1)(t_0)$. It must be $A_1(u_1, v_1)(t_0) < 0 = u_1(t_0)$. Let (t_1, t_2) be the maximal interval and contains t_0 such that $A_1(u_1, v_1)(t) < 0$ for all $t \in (t_1, t_2)$. Obviously, $(t_1, t_2) \neq [0, 1]$ by (H_3). If $t_2 < 1$, then $u_1(t) \equiv 0$ for $t \in [t_1, t_2]$, and $A_1(u_1, v_1)(t) < 0$ for $t \in (t_1, t_2)$, and $A_1(u_1, v_1)(t_2) = 0$. Thus, $A_1(u_1, v_1)'(t_2) = 0$. From (H_3) we get $(\phi_{p_1}(A_1(u_1, v_1)')(t))' = -f(0, v) \leq 0$ for $t \in [t_1, t_2]$, which implies that $A_1(u_1, v_1)'(t)$ is decrease on $[t_1, t_2]$. So $A_1(u_1, v_1)'(t) \geq 0$ for $t \in [t_1, t_2]$. Hence $A_1(u_1, v_1)(t) < 0$ and is bounded away from 0 everywhere in (t_1, t_2). This forces $t_1 = 0$ and $A_1(u_1, v_1)(0) < 0, A_1(u_1, v_1)'(0) \geq 0$. Thus, $\phi_{p_1}(A_1(u_1, v_1)(0)) < 0, \phi_{p_1}(A_1(u_1, v_1)'(0)) \geq 0$. On the other hand, by boundary value condition we have $\phi_{p_1}(A_1(u_1, v_1)(0)) = \frac{\beta_1}{\alpha_1}\phi_{p_1}(A_1(u_1, v_1)'(0))$ and so $\phi_{p_1}(A_1(u_1, v_1)(0)) \geq 0 > \phi_{p_1}(A_1(u_1, v_1)(0))$, which is impossible. If $t_1 > 0$, similar to the above, we have $1 \in (t_1, t_2)$, $A_1(u_1, v_1)(t_1) = 0$ and $A_1(u_1, v_1)'(t) < 0$ for $t \in (t_1, t_2)$. Hence $A_1(u_1, v_1)(t)$ is strictly decreasing on (t_1, t_2). So we have $A_1(u_1, v_1)(1) < 0, A_1(u_1, v_1)'(1) < 0$. Thus, $\phi_{p_1}(A_1(u_1, v_1)(1)) < 0, \phi_{p_1}(A_1(u_1, v_1)'(1)) < 0$. In fact, by boundary value condition we have $\phi_{p_1}(A_1(u_1, v_1)(1)) = -\frac{\delta_1}{\gamma_1}\phi_{p_1}(A_1(u_1, v_1)'(1))$ and so $\phi_{p_1}(A_1(u_1, v_1)(1)) > 0 > \phi_{p_1}(A_1(u_1, v_1)(1))$, which is a contradiction. In a word, we have $u_1 = A_1(u_1, v_1)$. Similarly, we can get $v_1 = A_2(u_1, v_1)$. Therefore, we conclude that (u_1, v_1) is a fixed point of A, and is also a solution of boundary value system (1.1) with $0 < ||(u_1, v_1)|| < a$.

Next, we need to show the existence of another fixed point of A. $\forall (u, v) \in \partial K'_a$, then $||(u, v)|| = a$ and $0 < u(t) + v(t) \leq a$, from (H_5) we have

$$\begin{aligned} ||F_1(u, v)|| &= F_1(u, v)(\sigma_1(u, v)) \\ &\leq \phi_{q_1}\left(\frac{\beta_1}{\alpha_1}\int_0^1 a_1(r)f^+(u(r), v(r))dr\right) \\ &+ \int_0^1 \phi_{q_1}\left(\int_s^{\sigma_1(u,v)} a_1(r)f^+(u(r), v(r))dr\right) ds \\ &< \frac{a}{M_1}\left[1 + \phi_{q_1}(\frac{\beta_1}{\alpha_1})\right]\left(\phi_{q_1}(\int_0^1 a_1(r)dr)ds\right) = \frac{a}{2}. \end{aligned}$$

Similarly, we get

$$||F_2(u, v)|| < \frac{a}{2}.$$

Thus,

$$||F(u, v)|| = ||F_1(u, v)|| + ||F_2(u, v)|| < \frac{a}{2} + \frac{a}{2} = a = ||(u, v)||.$$

It follows from Theorem 2.1 that

$$i(F, K'_a, K') = 1.$$

$\forall (u, v) \in \partial K'_b$, then $||(u, v)|| = b$. By Lemma 2.3, we have $\eta b \leq u(t) + v(t) \leq b$, for $t \in [\eta, 1 - \eta]$. From (H_6), we shall discuss it from three perspectives.

(i) If $\sigma_{1(u,v)} \in [\eta, 1-\eta]$, by Lemma 2.1, we have

$$\begin{aligned}
2\|F_1(u,v)\| &= 2F_1(u,v)(\sigma_{1(u,v)}) \\
&\geq \int_0^{\sigma_{1(u,v)}} \phi_{q_1}\left(\int_s^{\sigma_{1(u,v)}} a_1(r) f^+(u(r),v(r))dr\right) ds \\
&\quad + \int_{\sigma_{1(u,v)}}^1 \phi_{q_1}\left(\int_{\sigma_{1(u,v)}}^s a_1(r) f^+(u(r),v(r))dr\right) ds \\
&\geq \frac{b}{N_1}\left(\int_\eta^{\sigma_{1(u,v)}} \phi_{q_1}\left(\int_s^{\sigma_{1(u,v)}} a_1(r)dr\right) ds\right) \\
&\quad + \frac{b}{N_1}\left(\int_{\sigma_{1(u,v)}}^{1-\eta} \phi_{q_1}\left(\int_{\sigma_{1(u,v)}}^s a_1(r)dr\right) ds\right) \\
&\geq \frac{b}{N_1} A_1(\sigma_{1(u,v)}) \geq \frac{b}{N_1} L > 2b.
\end{aligned}$$

(ii) If $\sigma_{1(u,v)} \in (1-\eta, 1]$, by Lemma 2.1, we have

$$\begin{aligned}
\|F_1(u,v)\| &= F_1(u,v)(\sigma_{1(u,v)}) \\
&\geq \int_0^{\sigma_{1(u,v)}} \phi_{q_1}\left(\int_s^{\sigma_{1(u,v)}} a_1(r) f^+(u(r),v(r))dr\right) ds \\
&\geq \int_\eta^{1-\eta} \phi_{q_1}\left(\int_s^{1-\eta} a_1(r) f^+(u(r),v(r))dr\right) ds \\
&\geq \frac{b}{N_1}\int_\eta^{1-\eta} \phi_{q_1}\left(\int_s^{1-\eta} a_1(r)dr\right) ds \\
&= \frac{b}{N_1} A_1(1-\eta) \geq \frac{b}{N_1} L > 2b > b.
\end{aligned}$$

(iii) If $\sigma_{1(u,v)} \in (0, \eta)$, by Lemma 2.1, we have

$$\begin{aligned}
\|F_1(u,v)\| &= F_1(u,v)(\sigma_{1(u,v)}) \\
&\geq \int_{\sigma_{1(u,v)}}^1 \phi_{q_1}\left(\int_{\sigma_{1(u,v)}}^s a_1(r) f^+(u(r),v(r))dr\right) ds \\
&\geq \int_\eta^{1-\eta} \phi_{q_1}\left(\int_\eta^s a_1(r) f^+(u(r),v(r))dr\right) ds \\
&\geq \frac{b}{N_1}\int_\eta^{1-\eta} \phi_{q_1}\left(\int_\eta^s a_1(r)dr\right) ds \\
&= \frac{b}{N_1} A_1(\eta) \geq \frac{b}{N_1} L > 2b > b.
\end{aligned}$$

So we have

$$||F_1(u,v)|| > b.$$

Similarly, we get

$$||F_2(u,v)|| > b.$$

Thus,

$$||F(u,v)|| = ||F_1(u,v)|| + ||F_2(u,v)|| > 2b > b = ||(u,v)||.$$

It follows from Theorem 2.1 that

$$i(F, K'_b, K') = 0.$$

Thus $i(F, K'_b \setminus K'_a, K') = -1$ and F has a fixed point (u_2, v_2) in $K'_b \setminus K'_a$.

Finally, we prove that (u_2, v_2) is also a fixed point of A in $K'_b \setminus K'_a$. We claim that $A(u,v) = F(u,v)$ for $(u,v) \in (K'_b \setminus K'_a) \cap \{(u,v) : F(u,v) = (u,v)\}$. In fact, for $(u_2, v_2) \in (K'_b \setminus K'_a) \cap \{(u,v) : F(u,v) = (u,v)\}$, it is clear that $u_2(\sigma_1(u,v)) + v_2(\sigma_2(u,v)) = ||(u_2,v_2)|| > a$. Using Lemma 2.3, we have

$$\min_{\eta \le t \le 1-\eta} (u_2(t) + v_2(t)) \ge \eta(u_2(\sigma_1(u,v)) + v_2(\sigma_2(u,v))) = \eta||(u_2,v_2)|| > \eta a > d.$$

Thus for $t \in [\eta, 1-\eta], d \le u_2(t) + v_2(t) \le b$. From (H_4), we know that $f^+(u_2,v_2) = f(u_2,v_2), g^+(u_2,v_2) = g(u_2,v_2)$. This implies that $A(u_2,v_2) = F(u_2,v_2)$ for $(u_2,v_2) \in (K'_b \setminus K'_a) \cap \{(u,v) : F(u,v) = (u,v)\}$. Hence (u_2,v_2) is also a fixed point of A in $K'_b \setminus K'_a$, which is also a solution of boundary value system (1.1) with $a < ||(u_2,v_2)|| < b$. Therefore, we can know boundary value system (1.1) has at least two positive solutions (u_1,v_1) and (u_2,v_2) such that

$$0 \le ||(u_1,v_1)|| < a < ||(u_2,v_2)|| < b.$$

The proof of Theorem 3.1 is completed.

4 The existence of infinitely many positive solutions

In this section, we will discuss the existence of infinitely many positive solutions. We suppose that

(H_2') There exists a sequence $\{t_i\}_{i=1}^{\infty}$ such that $t_{i+1} < t_i$, $t_1 < 1/2$, $\lim_{i\to\infty} t_i = t^* \ge 0$, $\lim_{t\to t_i} a_i(t) = \infty$ $(i = 1, 2, \cdots)$, and

$$0 < \int_0^1 a_i(t)dt < \infty, \quad i = 1, 2.$$

It is easy to check that condition (H_2') implies that

$$0 < \int_0^1 \phi_i \left(\int_0^s a_i(r)dr \right) ds < +\infty, \ i = 1, 2.$$

Theorem 4.1. Suppose that conditions (H_1), (H_2') and (H_3) hold. Let $\{\eta_k\}_{k=1}^{\infty}$ be such that $\eta_k \in (t_{k+1}, t_k)$ $(k = 1, 2, \cdots)$, and let $\{a_k\}_{k=1}^{\infty}, \{b_k\}_{k=1}^{\infty}, \{d_k\}_{k=1}^{\infty}$ be such that

$$0 < \frac{d_k}{\eta_k} < a_k < \eta_k b_k < b_k, \ k = 1, 2, \cdots.$$

Furthermore, for each natural number k we assume that f, g satisfy the following conditions

(H_7): $f(u,v) \ge 0$, $g(u,v) \ge 0$, for $u + v \in [d_k, b_k]$;

(H_8): $f(u,v) < \phi_1(\frac{a_k}{M_1})$, $g(u,v) < \phi_2(\frac{a_k}{M_2})$, for $u + v \in [0, a_k]$;

(H_9): $\quad f(u,v) > \phi_1(\dfrac{b_k}{N_1}),\ g(u,v) > \phi_2(\dfrac{b_k}{N_2}),\quad$ for $u+v \in [\eta_k b_k, b_k]$.

Then, the boundary value system (1.1) has infinitely many solutions (u_k, v_k) such that $a_k < ||(u_k, v_k)|| < b_k,\ k = 1, 2, \cdots$.

Proof. Because $t^* < t_{k+1} < \eta_k < t_k < \frac{1}{2}$ $(k = 1, 2, \cdots)$, for any natural number k and $u \in K'$, by Lemma 2.3, we have

$$u(t) \geq \eta_k ||u||,\ t \in [\eta_k, 1-\eta_k].$$

We define two open subset sequences $\{K'_{a_k}\}_{k=1}^{\infty}$ and $\{K'_{b_k}\}_{k=1}^{\infty}$ of K' by

$$K'_{a_k} = \{u \in K' : \|u\| < a_k\},\ K'_{b_k} = \{u \in K' : \|u\| < b_k\},\ k = 1, 2, \cdots.$$

For a fixed natural number k and $\forall (u,v) \in \partial K'_{a_k}$, then $||(u,v)|| = a_k$ and $0 < u(t)+v(t) \leq a_k$, from (H_8) we have

$$\begin{aligned}
||F_1(u,v)|| &= F_1(u,v)(\sigma_1(u,v)) \\
&\leq \phi_{q_1}\left(\frac{\beta_1}{\alpha_1}\int_0^1 a_1(r)f^+(u(r),v(r))dr\right) \\
&+ \int_0^1 \phi_{q_1}\left(\int_s^{\sigma_{1(u,v)}} a_1(r)f^+(u(r),v(r))dr\right) ds \\
&< \frac{a_k}{M_1}\left[1+\phi_{q_1}(\frac{\beta_1}{\alpha_1})\right]\left(\phi_{q_1}(\int_0^1 a_1(r)dr)ds\right) = \frac{a_k}{2}.
\end{aligned}$$

Similarly, we get

$$||F_2(u,v)|| < \frac{a_k}{2}.$$

Thus,

$$||F(u,v)|| = ||F_1(u,v)|| + ||F_2(u,v)|| < \frac{a_k}{2} + \frac{a_k}{2} = a_k = ||(u,v)||.$$

It follows from Theorem 2.1 that

$$i(F, K'_{a_k}, K') = 1.$$

$\forall (u,v) \in \partial K'_{b_k}$, then $||(u,v)|| = b_k$. Using Lemma 2.3, we have $\eta_k b_k \leq u(t)+v(t) \leq b_k$ for $t \in [\eta_k, 1-\eta_k]$. Note that $[t_1, 1-t_1] \subseteq [\eta_k, 1-\eta_k]$. We discuss it from the following three ranges.

(i) If $\sigma_{1(u,v)} \in [t_1, 1-t_1]$, by Lemma 2.1 and condition (H_9), we have

$$\begin{aligned}
2\|F_1(u,v)\| &= 2F_1(u,v)(\sigma_{1(u,v)}) \\
&\geq \int_0^{\sigma_{1(u,v)}} \phi_{q_1}\left(\int_s^{\sigma_{1(u,v)}} a_1(r)f^+(u(r),v(r))dr\right) ds \\
&\quad + \int_{\sigma_{1(u,v)}}^1 \phi_{q_1}\left(\int_{\sigma_{1(u,v)}}^s a_1(r)f^+(u(r),v(r))dr\right) ds \\
&\geq \frac{b_k}{N_1}\left(\int_{t_1}^{\sigma_{1(u,v)}} \phi_{q_1}\left(\int_s^{\sigma_{1(u,v)}} a_1(r)dr\right) ds\right) \\
&\quad + \frac{b_k}{N_1}\left(\int_{\sigma_{1(u,v)}}^{1-t_1} \phi_{q_1}\left(\int_{\sigma_{1(u,v)}}^s a_1(r)dr\right) ds\right) \\
&\geq \frac{b_k}{N_1}A_1(\sigma_{1(u,v)}) \geq \frac{b_k}{N_1}L > 2b_k.
\end{aligned}$$

(ii) If $\sigma_{1(u,v)} \in (1-t_1, 1]$, by Lemma 2.1 and condition (H_9), we have

$$\begin{aligned}\|F_1(u,v)\| &= F_1(u,v)(\sigma_{1(u,v)}) \\ &\geq \int_0^{\sigma_{1(u,v)}} \phi_{q_1}\left(\int_s^{\sigma_{1(u,v)}} a_1(r) f^+(u(r), v(r)) dr\right) ds \\ &\geq \int_{t_1}^{1-t_1} \phi_{q_1}\left(\int_s^{1-t_1} a_1(r) f^+(u(r), v(r)) dr\right) ds \\ &\geq \frac{b_k}{N_1} \int_{t_1}^{1-t_1} \phi_{q_1}\left(\int_s^{1-t_1} a_1(r) dr\right) ds \\ &= \frac{b_k}{N_1} A_1(1-t_1) \geq \frac{b_k}{N_1} L > 2b_k > b_k.\end{aligned}$$

(iii) If $\sigma_{1(u,v)} \in (0, t_1)$, by Lemma 2.1 and condition (H_9), we have

$$\begin{aligned}\|F_1(u,v)\| &= F_1(u,v)(\sigma_{1(u,v)}) \\ &\geq \int_{\sigma_{1(u,v)}}^1 \phi_{q_1}\left(\int_{\sigma_{1(u,v)}}^s a_1(r) f^+(u(r), v(r)) dr\right) ds \\ &\geq \int_{t_1}^{1-t_1} \phi_{q_1}\left(\int_\eta^s a_1(r) f^+(u(r), v(r)) dr\right) ds \\ &\geq \frac{b_k}{N_1} \int_{t_1}^{1-t_1} \phi_{q_1}\left(\int_{t_1}^s a_1(r) dr\right) ds \\ &= \frac{b_k}{N_1} A_1(t_1) \geq \frac{b_k}{N_1} L > 2b_k > b_k.\end{aligned}$$

So we have

$$||F_1(u,v)|| > b_k.$$

Similarly, we get

$$||F_2(u,v)|| > b_k.$$

Thus,

$$||F(u,v)|| = ||F_1(u,v)|| + ||F_2(u,v)|| > b_k + b_k = 2b_k > b_k = ||(u,v)||.$$

It follows from Theorem 2.1 that

$$i(F, K'_{b_k}, K') = 0.$$

Thus $i(F, K'_{b_k} \setminus K'_{a_k}, K') = -1$ and F has a fixed point (u_k, v_k) in $K'_{b_k} \setminus K'_{a_k}$.

Finally, we prove that (u_k, v_k) is also a fixed point of A in $K'_{b_k} \setminus K'_{a_k}$. We claim that $A(u,v) = F(u,v)$ for $(u,v) \in (K'_{b_k} \setminus K'_{a_k}) \cap \{(u,v) : F(u,v) = (u,v)\}$. In fact, for $(u_k, v_k) \in (K'_{b_k} \setminus K'_{a_k}) \cap \{(u,v) : F(u,v) = (u,v)\}$, it is clear that $u_k(\sigma_1(u,v)) + v_k(\sigma_2(u,v)) = ||(u_k, v_k)|| > a_k$. By Lemma 2.3, we have

$$\min_{\eta_k \leq t \leq 1-\eta_k} (u_k(t) + v_k(t)) \geq \eta_k(u_k(\sigma_1(u,v)) + v_k(\sigma_2(u,v))) = \eta_k||(u_k, v_k)|| > \eta_k a_k > d_k.$$

Thus for $t \in [\eta_k, 1-\eta_k], d_k \leq u_k(t) + v_k(t) \leq b_k$. From (H_7), we know that $f^+(u_k, v_k) = f(u_k, v_k), g^+(u_k, v_k) = g(u_k, v_k)$. This implies that $A(u_k, v_k) = F(u_k, v_k)$ for $(u_k, v_k) \in (K'_{b_k} \setminus$

$K'_{a_k}) \cap \{(u,v) : F(u,v) = (u,v)\}$. Hence (u_k, v_k) is also a fixed point of A in $K'_{b_k} \setminus K'_{a_k}$, which is also a solution of boundary value system (1.1) with $a_k < ||(u_k, v_k)|| < b_k$. Therefore, by the arbitrary of k, we can know boundary value system (1.1) has infinitely many solutions (u_k, v_k) such that $a_k < ||(u_k, v_k)|| < b_k$ $k = 1, 2, \cdots$. The proof of Theorem 4.1 is completed.

5 Remarks

In the section, we present some remarks as follows.

Remark5.1.[See 11] We can provide an function $a(t)$ satisfying condition (H_2'). In fact, let

$$\Delta = \sqrt{2}\left(\frac{\pi^2}{3} - \frac{9}{4}\right), \quad t_0 = \frac{5}{16}, \quad t_n = t_0 - \sum_{i=1}^{n-1} \frac{1}{(i+2)^4}, \quad n = 1, 2, \cdots.$$

Consider function $a(t) : [0,1] \to (0, +\infty)$ given by $a(t) = \sum\limits_{n=1}^{\infty} a_n(t), \quad t \in [0,1]$, where

$$a_n(t) = \begin{cases} \frac{1}{n(n+1)(t_{n+1}+t_n)}, & 0 \le t < \frac{t_{n+1}+t_n}{2}, \\ \frac{1}{\Delta(t_n - t)^{\frac{1}{2}}}, & \frac{t_{n+1}+t_n}{2} \le t < t_n, \\ \frac{1}{\Delta(t - t_n)^{\frac{1}{2}}}, & t_n \le t \le \frac{t_{n-1}+t_n}{2}, \\ \frac{2}{n(n+1)(2 - t_n - t_{n-1})}, & \frac{t_{n-1}+t_n}{2} < t \le 1. \end{cases}$$

It is easy to know $t_1 = \frac{1}{4} < \frac{1}{2}, \quad t_n - t_{n+1} = \frac{1}{(n+2)^4}$ $(n = 1, 2, \cdots)$, and

$$t^* = \lim_{n\to\infty} t_n = \frac{5}{16} - \sum_{i=1}^{\infty} \frac{1}{(i+2)^4} = \frac{21}{16} - \frac{\pi^4}{90} > \frac{1}{5},$$

where $\sum\limits_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$. From $\sum\limits_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, we have

$$\begin{aligned} \sum_{n=1}^{\infty} \int_0^1 a_n(t)dt &= \sum_{n=1}^{\infty} \frac{2}{n(n+1)} + \frac{1}{\Delta} \sum_{n=1}^{\infty} \left[\int_{\frac{t_{n+1}+t_n}{2}}^{t_n} \frac{1}{(t_n - t)^{\frac{1}{2}}} dt + \int_{t_n}^{\frac{t_n + t_{n-1}}{2}} \frac{1}{(t - t_n)^{\frac{1}{2}}} dt \right] \\ &= 2 + \frac{\sqrt{2}}{\Delta} \sum_{n=1}^{\infty} \left[(t_n - t_{n+1})^{\frac{1}{2}} + (t_{n-1} - t_n)^{\frac{1}{2}} \right] \\ &= 2 + \frac{\sqrt{2}}{\Delta} \sum_{n=1}^{\infty} \left[\frac{1}{(n+2)^2} + \frac{1}{(n+1)^2} \right] \\ &= 2 + \frac{\sqrt{2}}{\Delta} \sum_{n=1}^{\infty} \left[(\frac{\pi^2}{6} - \frac{5}{4}) + (\frac{\pi^2}{6} - 1) \right] \\ &= 2 + \frac{\sqrt{2}}{\Delta} [\frac{\pi^2}{3} - \frac{9}{4}] = 3. \end{aligned}$$

Hence

$$\int_0^1 a(t)dt = \int_0^1 \sum_{n=1}^{\infty} a_n(t)dt = \sum_{n=1}^{\infty} \int_0^1 a_n(t)dt < \infty,$$

which implies that condition (H_2') holds.

References

1 C. Bandle, M. Kwong, Semilinear elliptic problems in annular domains. J. Appl. Math. Phys. ZAMP, 40 (1989) 245-257.

2 R. Agarwal, H. Lü, D. O'Regan, Existence theorems for the one-dimensional singular p-Laplacian equation with sign changing nonlinearities, Appl. Math. Comput. 143 (2003) 15-38.

3 K. Deimling, Nonlinear Functional Analysis. Berlin: Springer-Verlag, 1980.

4 D. Guo and V. Lakshmikantham, Nonlinear Problems in Abstract Cone. Academic Press, San Diego, 1988.

5 D. Guo, V. Lakshmikantham and X. Liu, Nonlinear Integral Equations in Abstract Spaces. Kluwer Academic Publishers, 1996.

6 J.A. Gatica, V. Oliker and P. Waltman, Singular boundary value problems for second order ordinary differential equation, J. Differential Equations 79 (1989) 62-78.

7 X. He, The existence of positive solutions of p-Laplacian equation, Acta. Math. Sinica, 46(4) (2003) 805-810.

8 D. Ji, M. Feng, W. Ge, Multiple positive solutions for multipoint boundary value problems with sign changing nonlinearity, Appl. Math. Comput. 196 (2008) 515-520.

9 D. Ji, Y. Tian, W. Ge, Positive solutions for one-dimensional p-Laplacian boundary value problems with sign changing nonlinearity, Nonlinear Anal. 71 (2009) 5406-5416.

10 E. Kaufmann, N. Kosmatov, A multiplicity results for a boundary value problem with infinitely many singularities, J. Math. Anal. Appl. 269 (2002) 444-453.

11 B. Liu, Positive solutions three-points boundary value problems for one-dimensional p-Laplacian with infinitely many singularities, Appl. Math. Lett. 17 (2004) 655-661.

12 R. Ma, Positive solutions of singular second order boundary value problems. Acta. Math. Sinica, 41(6) (1998) 1225-1230 (in Chinese).

13 H. Su, Z. Wei, F. Xu, The existence of positive solutions for nonlinear singular boundary value system with p-Laplacian, Appl. Math. Comput. 181(2) (2006) 826-836.

14 H. Su, Z. Wei, F. Xu, The existence of countably many positive solutions for a system of nonlinear singular boundary value problems with the p-Laplacian operator, J. Math. Anal. Appl. 325(1) (2007) 319-332.

15 H. Wang, On the existence of positive solutions for semilinear elliptic equations in the annulus. J. Differential Equations 109 (1994) 1-7.

16 Z. Wei, Positive solutions of singular Dirichlet boundary value problems. Chinese Annals of Mathematics, 20(A) (1999) 543-552 (in Chinese).

17 F. Wong, The existence of positive solutions for m-Laplacian BVPs, Appl. Math. Lett. 12 (1999) 12-17.

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