Electronic Journal of Qualitative Theory of Differential Equations
2015, No. 50, 1–3; doi:10.14232/ejqtde.2015.1.50 http://www.math.u-szeged.hu/ejqtde/

Corrigendum to "Weighted L^p estimates for the elliptic Schrödinger operator" [*Electron. J. Qual. Theory Differ. Equ.* 2014, No. 33, 1–13]

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Received 25 June 2015, appeared 14 August 2015
Communicated by Patrizia Pucci

Abstract. An error in Lemma 1.5 in "Weighted L^p estimates for the elliptic Schrödinger operator" [*Electron. J. Qual. Theory Differ. Equ.* 2014, No. 33, 1–13] is corrected.

Keywords: weighted, regularity, L^p estimates, elliptic, Schrödinger operator.

2010 Mathematics Subject Classification: 35J10, 35J15.

The author regrets to inform that Lemma 1.5 in [1] is not fully correct. Since the main results in [1] eventually rely on this lemma, we shall give the following new lemma instead of the wrong Lemma 1.5 in [1] and then put it behind Lemma 1.9 in [1].

Lemma 1.9. *Assume that $w \in A_q$ for some $q > 1$.*

(1) There exists a positive constant $q_1 \in (1, q)$ such that

$$w \in A_{q_1}.$$

(2) Let $q_2 = \frac{q}{q_1} \in (1, q)$. Then we have

$$L^q_w(B_r) \subset L^{q_2}(B_r).$$

Proof. Since $w \in A_q$, from Definition 1.3 in [1] we have

$$\begin{aligned}
&\left(\fint_{B_r} w(x)\,dx\right)\left(\fint_{B_r} w(x)^{\frac{-1}{q-1}}\,dx\right)^{q-1}\\
&\quad= \left(\fint_{B_r} \left(w(x)^{\frac{-1}{q-1}}\right)^{1-q} dx\right)\left(\fint_{B_r} w(x)^{\frac{-1}{q-1}}\,dx\right)^{q-1}\\
&\quad= \left[\left(\fint_{B_r} \left(w(x)^{\frac{-1}{q-1}}\right)^{-\frac{1}{\frac{q}{q-1}-1}} dx\right)^{\frac{q}{q-1}-1}\left(\fint_{B_r} w(x)^{\frac{-1}{q-1}}\,dx\right)\right]^{q-1} \le C \qquad (1.1)
\end{aligned}$$

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for any balls B_r in $\mathbb{R}^n$, which implies that $w(x)^{\frac{-1}{q-1}} \in A_{\frac{q}{q-1}}$. Therefore, from Lemma 1.8 in [1] we have

$$\left(\fint_{B_r} w(x)^{-\frac{1+\epsilon_0'}{q-1}}\, dx\right)^{\frac{1}{1+\epsilon_0'}} \leq C \fint_{B_r} w(x)^{\frac{-1}{q-1}}\, dx \tag{1.2}$$

for some $\epsilon_0' \in (0,1)$. Let

$$q_1 = 1 + \frac{q-1}{1+\epsilon_0'} \in (1,q).$$

Then from (1.2) and the fact that $w \in A_q$ we find that

$$\begin{aligned}
&\left(\fint_{B_r} w(x)\, dx\right)\left(\fint_{B_r} w(x)^{\frac{-1}{q_1-1}} dx\right)^{q_1-1} \\
&= \left(\fint_{B_r} w(x)\, dx\right)\left(\fint_{B_r} w(x)^{-\frac{1+\epsilon_0'}{q-1}} dx\right)^{\frac{q-1}{1+\epsilon_0'}} \\
&\leq C\left(\fint_{B_r} w(x)\, dx\right)\left(\fint_{B_r} w(x)^{-\frac{1}{q-1}} dx\right)^{q-1} \leq C,
\end{aligned} \tag{1.3}$$

which implies that $w \in A_{q_1}$. Furthermore, if $f \in L^q_w(B_r)$, then from Hölder's inequality and (1.3) we have

$$\begin{aligned}
\int_{B_r} |f|^{\frac{q}{q_1}}\, dx &= \int_{B_r} |f|^{\frac{q}{q_1}}\, w(x)^{\frac{1}{q_1}} w(x)^{-\frac{1}{q_1}}\, dx \\
&\leq \left(\int_{B_r} |f|^q\, w(x)\, dx\right)^{\frac{1}{q_1}} \left(\int_{B_r} w(x)^{-\frac{1}{q_1-1}} dx\right)^{1-\frac{1}{q_1}} \\
&\leq C\left(\int_{B_r} |f|^q\, w(x)\, dx\right)^{\frac{1}{q_1}} \left(\frac{|B_r|}{w(B_r)}\right)^{\frac{1}{q_1}} \leq C,
\end{aligned}$$

since $w \in L^1_{\mathrm{loc}}(\mathbb{R}^n)$ and $w > 0$ almost everywhere. This finishes our proof by choosing $q_2 = \frac{q}{q_1} \in (1,q)$. □

We shall add the following sentences in front of "Next, we shall prove the following important result" in [1], page 5, line -6:

"Assume that $w \in A_p$. Then from Lemma 1.9 (1) (see above) we find that

$$w \in A_{p_1} \quad \text{for some} \quad p_1 \in (1,p)." \tag{1.4}$$

Moreover, we shall change "Assume that $1 < q < p$" in Lemma 2.2 of [1] to "Assume that $w \in A_p$" and add the sentence "where $q = \frac{p}{p_1} \in (1,p)$ and p_1 is defined in (1.4)," behind (2.2) in [1]. Furthermore, we shall change "Assume that $1 < q < p$ and $w \in A_p$" in Corollary 2.3 of [1] to "Assume that $w \in A_p$" and add the sentence "where $q = \frac{p}{p_1} \in (1,p)$ and p_1 is defined in (1.4)," behind (2.8) in [1].

The author would like to apologize for any inconvenience caused.

Acknowledgments

This work is supported in part by the NSFC (11471207) and the Innovation Program of Shanghai Municipal Education Commission (14YZ027).

References

[1] F. Yao, Weighted L^p estimates for the elliptic Schrödinger operator, *Electron. J. Qual. Theory Differ. Equ.* **2014**, No. 33, 1–13. MR3250024; url