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Global dynamic behaviors for a delayed Nicholson's blowflies model with a linear harvesting term*

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Abstract

In this paper, we study a generalized Nicholson's blowflies model with a linear harvesting term, which is defined on the positive function space. Under proper conditions, we employ a novel proof to establish some criteria for the global dynamic behaviors on existence of positive solutions, permanence, and exponential stability of the zero equilibrium point for this model. Moreover, we give two examples and their numerical simulations to illustrate our main results.

Keywords: Nicholson's blowflies model; linear harvesting term; positive solution; permanence; exponential stability.

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1. Introduction

Recently, assuming that a harvesting term is a function of the delayed estimate for the true population, L. Berezansky et al. [1] proposed the following Nicholson's blowflies model

$$x'(t) = -\delta x(t) + px(t-\tau)e^{-ax(t-\tau)} - Hx(t-\sigma),\ \delta, p, \tau, a, H, \sigma \in (0,\ +\infty), \tag{1.1}$$

where $Hx(t-\sigma)$ is the linear harvesting term, $x(t)$ is the size of the population at time t, p is the maximum per capita daily egg production, $\frac{1}{a}$ is the size at which the population

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reproduces at its maximum rate, δ is the per capita daily adult death rate, and τ is the generation time. Moreover, L. Berezansky et al. [1] formulated an open problem: How about the dynamic behaviors of (1.1). Consequently, some criteria were established in [2−5] to guarantee the existence of positive periodic solutions for (1.1) and its generalized equations by applying the method of coincidence degree; some sufficient conditions were also obtained in [6−8] to ensure that the solutions of its generalized system converge locally exponentially to a positive almost periodic solution. However, it is difficult to study the global dynamic behaviors of the Nicholson's blowflies model with a linear harvesting term. So far, there is no literature considering the global existence of positive solutions and the global permanence for (1.1). In particular, there is no research on the global stability of the zero equilibrium point of (1.1). Thus, it is also a unsolved open problem to reveal the global dynamic behaviors of Nicholson's blowflies model (1.1).

Motivated by the above discussions, the main purpose of this paper is to establish some criteria for the global dynamic behaviors on existence of positive solutions, permanence, and exponential stability of zero equilibrium point for Nicholson's blowflies model with a linear harvesting term. Since the coefficients and delays in differential equations of population and ecology problems are usually time-varying in the real world, we consider the following Nicholson's blowflies model with a linear harvesting term

$$\begin{aligned} x'(t) &= -a(t)x(t) + \sum_{j=2}^{m} \beta_j(t)x(t-\tau_j(t))e^{-\gamma_j(t)x(t-\tau_j(t))} \\ &\quad + \beta_1(t)x(t-\tau_1(t))e^{-\gamma_1(t)x(t)} - H(t)x(t-\sigma(t)), \end{aligned} \tag{1.2}$$

where $a(t)$, $H(t)$, $\sigma(t)$ and $\gamma_j(t)$ are continuous functions bounded above and below by positive constants, $\beta_j(t)$ and $\tau_j(t)$ are nonnegative bounded continuous functions, and $j = 1, 2, \cdots, m$. Obviously, (1.1) is a special case of (1.2) with constant coefficients and delays.

For convenience, we introduce some notations. In the following part of this paper, given a bounded continuous function g defined on R, let g^+ and g^- be defined as

$$g^+ = \sup_{t\in R} g(t), \quad g^- = \inf_{t\in R} g(t).$$

It will be assumed that

$$r := \max\{\max_{1\le j\le m} \tau_j^+, \quad \sigma^+\}. \tag{1.3}$$

Throughout this paper, let $C = C([-r,\ 0], R)$ be the continuous functions space equipped with the usual supremum norm $||\cdot||$, and let $C_+ = C([-r,\ 0], (0,\ +\infty))$. If $x(t)$ is continuous and defined on $[-r+t_0,\ \sigma)$ with $t_0, \sigma \in R$, then we define $x_t \in C$ where $x_t(\theta) = x(t+\theta)$ for all $\theta \in [-r,\ 0]$.

Due to the biological interpretation of model (1.2), only positive solutions are meaningful and therefore admissible. Thus we just consider admissible initial conditions

$$x_{t_0} = \varphi, \quad \varphi \in C_+. \tag{1.4}$$

Define a continuous map $f : R \times C_+ \to R$ by setting

$$f(t,\varphi) = -a(t)\varphi(0) + \sum_{j=2}^{m} \beta_j(t)\varphi(-\tau_j(t))e^{-\gamma_j(t)\varphi(-\tau_j(t))}$$

$$+\beta_1(t)\varphi(-\tau_1(t))e^{-\gamma_1(t)\varphi(0)} - H(t)\varphi(-\sigma(t)).$$

Then, f is a locally Lipschitz map with respect to $\varphi \in C_+$, which ensures the existence and uniqueness of the solution of (1.2) with admissible initial conditions (1.4).

We write $x_t(t_0,\ \varphi)(x(t;t_0,\varphi))$ for an admissible solution of the admissible initial value problem (1.2) and (1.4). Also, let $[t_0,\ \eta(\varphi))$ be the maximal right-interval of existence of $x_t(t_0,\varphi)$.

2. Global existence of the positive solutions

In this section, we establish sufficient conditions on the global existence of the positive solutions for (1.2).

Theorem 2.1. Assume that

$$\inf_{t\in R}\{\beta_1(t) - H(t)\} > 0, \quad \text{and} \quad \tau_1(t) \equiv \sigma(t) \ \text{ for all } \ t \in R. \tag{2.1}$$

Then, the solution $x_t(t_0,\varphi) \in C_+$ for all $t \in [t_0,\ \eta(\varphi))$, the set of $\{x_t(t_0,\varphi) : t \in [t_0,\ \eta(\varphi))\}$ is bounded, and $\eta(\varphi) = +\infty$.

Proof. We first show that

$$x(t) > 0, \quad \text{for all} \ \ t \in (t_0,\ \eta(\varphi)). \tag{2.2}$$

Suppose, for the sake of contradiction, that (2.2) does not hold. Then, there exists $t_1 \in (t_0,\ \eta(\varphi))$ such that

$$x(t_1) = 0 \ \text{ and } \ x(t) > 0 \ \text{ for all } \ t \in [t_0 - r,\ t_1). \tag{2.3}$$

From (1.2) and (2.1), (2.3) leads to

$$\begin{aligned}
0 &\geq x'(t_1)\\
&= -a(t_1)x(t_1)+\sum_{j=2}^{m}\beta_j(t_1)x(t_1-\tau_j(t_1))e^{-\gamma_j(t_1)x(t_1-\tau_j(t_1))}\\
&\quad +\beta_1(t_1)x(t_1-\tau_1(t_1))e^{-\gamma_1(t_1)x(t_1)}-H(t_1)x(t_1-\sigma(t_1))\\
&= \sum_{j=2}^{m}\beta_j(t_1)x(t_1-\tau_j(t_1))e^{-\gamma_j(t_1)x(t_1-\tau_j(t_1))}\\
&\quad +\beta_1(t_1)x(t_1-\tau_1(t_1))-H(t_1)x(t_1-\tau_1(t_1))\\
&\geq x(t_1-\tau_1(t_1))[\beta_1(t_1)-H(t_1)]\\
&> 0,
\end{aligned}$$

which is a contradiction and implies that (2.2) holds.

For each $t\in[t_0-r,\ \eta(\varphi))$, we define

$$M(t)=\max\{\xi:\xi\leq t, x(\xi)=\max_{t_0-r\leq s\leq t}x(s)\}.$$

We now show that $x(t)$ is bounded on $[t_0,\ \eta(\varphi))$. In the contrary case, observe that $M(t)\to\eta(\varphi)$ as $t\to\eta(\varphi)$, we have

$$\lim_{t\to\eta(\varphi)}x(M(t))=+\infty. \tag{2.4}$$

But $x(M(t))=\max_{t_0-r\leq s\leq t}x(s)$, and so $x'(M(t))\geq 0$, for all $M(t)\geq t_0$. Thus,

$$\begin{aligned}
0 &\leq x'(M(t))\\
&= -a(M(t))x(M(t))\\
&\quad +\sum_{j=2}^{m}\beta_j(M(t))x(M(t)-\tau_j(M(t)))e^{-\gamma_j(M(t))x(M(t)-\tau_j(M(t)))}\\
&\quad +\beta_1(M(t))x(M(t)-\tau_1(M(t)))e^{-\gamma_1(M(t))x(M(t))}\\
&\quad -H(M(t))x(M(t)-\sigma(M(t))),\quad \text{for all}\quad M(t)\geq t_0,
\end{aligned}$$

which, together with (2.2) and the fact that $\sup_{u\geq 0}ue^{-u}=\frac{1}{e}$, yields

$$\begin{aligned}
&x(M(t))\\
\leq\ & \sum_{j=2}^{m}\frac{\beta_j(M(t))}{\gamma_j(M(t))a(M(t))}\gamma_j(M(t))x(M(t)-\tau_j(M(t)))e^{-\gamma_j(M(t))x(M(t)-\tau_j(M(t)))}\\
&+\frac{\beta_1(M(t))}{\gamma_1(M(t))a(M(t))}\gamma_1(M(t))x(M(t)-\tau_1(M(t)))e^{-\gamma_1(M(t))x(M(t))}
\end{aligned}$$

$$\leq \sum_{j=2}^{m} \frac{\beta_j(M(t))}{\gamma_j(M(t))a(M(t))}\frac{1}{e} + \frac{\beta_1(M(t))}{\gamma_1(M(t))a(M(t))}\gamma_1(M(t))x(M(t))e^{-\gamma_1(M(t))x(M(t))}$$

$$\leq \sum_{j=1}^{m} \frac{\beta_j(M(t))}{\gamma_j(M(t))a(M(t))}\frac{1}{e}, \quad \text{where} \quad M(t) \geq t_0. \tag{2.5}$$

Letting $t \to \eta(\varphi)$, (2.4) and (2.5) imply a contradiction. This implies that $x(t)$ is bounded on $[t_0, \ \eta(\varphi))$. From Theorem 2.3.1 in [9], we easily obtain $\eta(\varphi) = +\infty$. This completes the proof of Theorem 2.1.

3. Global permanence

In this section, we shall derive new sufficient conditions for checking the global permanence of model (1.2).

Theorem 3.1. Suppose that all conditions in Theorem 2.1 are satisfied. Let

$$\liminf_{t\to+\infty} \left\{ \sum_{j=2}^{m} \frac{\beta_j(t)}{a(t)} + [\frac{\beta_1(t)}{a(t)} - \frac{H(t)}{a(t)}] \right\} > 1. \tag{3.1}$$

Then model (1.2) is permanent, i.e., there exist two positive constants k and K such that

$$k \leq \liminf_{t\to+\infty} x(t) \leq \limsup_{t\to+\infty} x(t) \leq K, \tag{3.2}$$

where $x(t) = x(t; t_0, \varphi)$.

Proof. From the proof of Theorem 2.1, we obtain that there exists a positive constant K such that

$$\limsup_{t\to+\infty} x(t) \leq K. \tag{3.3}$$

We next prove that there exists a positive constant l such that

$$\liminf_{t\to+\infty} x(t) = l. \tag{3.4}$$

Otherwise, we assume that $\liminf_{t\to+\infty} x(t) = 0$. For each $t \geq t_0$, we define

$$m(t) = \max\{\xi : \xi \leq t, x(\xi) = \min_{t_0\leq s\leq t} x(s)\}.$$

Observe that $m(t) \to +\infty$ as $t \to +\infty$ and that

$$\lim_{t\to+\infty} x(m(t)) = 0. \tag{3.5}$$

Thus, (2.1) implies that there exists a constant $T_1 > t_0 + r$ such that

$$\beta_1(m(t))e^{-\gamma_1(m(t))x(m(t))} - H(m(t)) > 0, \quad \text{for all} \quad m(t) > T_1. \tag{3.6}$$

However, $x(m(t)) = \min_{t_0 \le s \le t} x(s)$, and so $x'(m(t)) \le 0$ for all $m(t) > t_0$. According to (1.2), we have

$$\begin{aligned} 0 &\ge x'(m(t)) \\ &= -a(m(t))x(m(t)) + \sum_{j=2}^{m} \beta_j(m(t))x(m(t) - \tau_j(m(t)))e^{-\gamma_j(m(t))x(m(t)-\tau_j(m(t)))} \\ &\quad + x(m(t) - \tau_1(m(t)))[\beta_1(m(t))e^{-\gamma_1(m(t))x(m(t))} - H(m(t))], \text{ where } m(t) > T_1. \end{aligned} \tag{3.7}$$

Consequently, (3.6) and (3.7) lead to

$$\begin{aligned} &a(m(t))x(m(t)) \\ \ge\ &\beta_j(m(t))x(m(t) - \tau_j(m(t)))e^{-\gamma_j(m(t))x(m(t)-\tau_j(m(t)))},\ j = 2, 3, \cdots, m, \end{aligned} \tag{3.8}$$

and

$$\begin{aligned} &a(m(t))x(m(t)) \\ \ge\ &x(m(t) - \tau_1(m(t)))[\beta_1(m(t))e^{-\gamma_1(m(t))x(m(t))} - H(m(t))], \end{aligned} \tag{3.9}$$

where $m(t) > T_1$. This, together with (3.5), implies that

$$\lim_{t \to +\infty} x(m(t) - \tau_j(m(t))) = 0,\ j = 1, 2, \cdots, m. \tag{3.10}$$

Noting that the continuities and boundedness of the functions $a(t)$, $H(t)$ and $\beta_j(t)$, we can select a sequence $\{t_n\}_{n=1}^{+\infty}$ such that $\lim_{n \to +\infty} t_n = +\infty$, and

$$\lim_{n \to +\infty} \frac{\beta_j(m(t_n))}{a(m(t_n))} = a_j^*, \qquad \lim_{n \to +\infty} \frac{H(m(t_n))}{a(m(t_n)} = H^*,\ j = 1, 2, \cdots, m. \tag{3.11}$$

In view of (3.7), for sufficiently large n, we get

$$\begin{aligned} &a(m(t_n)) \\ \ge\ &\sum_{j=2}^{m} \beta_j(m(t_n)) \frac{x(m(t_n) - \tau_j(m(t_n)))e^{-\gamma_j(m(t_n))x(m(t_n)-\tau_j(m(t_n)))}}{x(m(t_n))} \\ &+ \frac{x(m(t_n) - \tau_1(m(t_n)))}{x(m(t_n)} [\beta_1(m(t_n))e^{-\gamma_1(m(t_n))x(m(t_n))} - H(m(t_n))] \\ \ge\ &\sum_{j=2}^{m} \beta_j(m(t_n)) \frac{x(m(t_n) - \tau_j(m(t_n)))e^{-\gamma_j(m(t_n))x(m(t_n)-\tau_j(m(t_n)))}}{x(m(t_n) - \tau_j(m(t_n)))} \\ &+ \frac{x(m(t_n) - \tau_1(m(t_n)))}{x(m(t_n) - \tau_1(m(t_n)))} [\beta_1(m(t_n))e^{-\gamma_1(m(t_n))x(m(t_n))} - H(m(t_n))] \\ =\ &\sum_{j=2}^{m} \beta_j(m(t_n))e^{-\gamma_j(m(t_n))x(m(t_n)-\tau_j(m(t_n)))} \\ &+ [\beta_1(m(t_n))e^{-\gamma_1(m(t_n))x(m(t_n))} - H(m(t_n))], \end{aligned}$$

and

$$1 \geq \sum_{j=2}^{m} \frac{\beta_j(m(t_n))}{a(m(t_n))} e^{-\gamma_j(m(t_n))x(m(t_n)-\tau_j(m(t_n)))}$$
$$+[\frac{\beta_1(m(t_n))}{a(m(t_n))} e^{-\gamma_1(m(t_n))x(m(t_n))} - \frac{H(m(t_n))}{a(m(t_n))}]. \quad (3.12)$$

Letting $n \to +\infty$, (3.11) and (3.12) yield that

$$\begin{aligned} 1 &\geq \sum_{j=2}^{m} \lim_{n\to+\infty} \frac{\beta_j(m(t_n))}{a(m(t_n))} \lim_{n\to+\infty} e^{-\gamma_j(m(t_n))x(m(t_n)-\tau_j(m(t_n)))} \\ &\quad + \lim_{n\to+\infty} [\frac{\beta_1(m(t_n))}{a(m(t_n))} e^{-\gamma_1(m(t_n))x(m(t_n))} - \frac{H(m(t_n))}{a(m(t_n))}] \\ &= \lim_{n\to+\infty} \left\{ \sum_{j=2}^{m} \frac{\beta_j(m(t_n))}{a(m(t_n))} + [\frac{\beta_1(m(t_n))}{a(m(t_n))} - \frac{H(m(t_n))}{a(m(t_n))}] \right\} \\ &\geq \liminf_{t\to+\infty} \left\{ \sum_{j=2}^{m} \frac{\beta_j(t)}{a(t)} + [\frac{\beta_1(t)}{a(t)} - \frac{H(t)}{a(t)}] \right\}, \end{aligned} \quad (3.13)$$

which contradicts to (3.1). Hence, (3.4) holds. This completes the proof of Theorem 3.1.

4. Global exponential stability for the zero equilibrium point

In this section, we establish sufficient conditions on the global exponential stability of the zero equilibrium point for (1.2).

Theorem 4.1. Suppose that all conditions in Theorem 2.1 are satisfied. Let

$$\max_{1\leq j\leq m} \gamma_j^+ \leq 1, \ \limsup_{t\to+\infty} \left\{ \sum_{j=2}^{m} \frac{\beta_j(t)}{a(t)} + [\frac{\beta_1(t)}{a(t)} - \frac{H(t)}{a(t)}] \right\} < 1. \quad (4.1)$$

Then 0 is a globally exponentially stable equilibrium point on C_+, i.e., there exist two constants $M > 0$ and $T > t_0$ such that

$$0 < x(t; t_0, \varphi) < Me^{-\lambda t} \ \text{ for all } \ t > T. \quad (4.2)$$

Proof. Let $x(t) = x(t; t_0, \varphi)$. In view of Theorem 2.1, the set of $\{x_t(t_0, \varphi) : t \in [t_0, +\infty)\}$ is bounded, and

$$0 < x(t) \quad \text{for all } \ t > t_0. \quad (4.3)$$

From (4.1), we obtain that there exist $T > t_0$ and $0 < \eta_0 < 1$ such that

$$\sum_{j=2}^{m} \frac{\beta_j(t)}{a(t)} + [\frac{\beta_1(t)}{a(t)} - \frac{H(t)}{a(t)}] < \eta_0 < 1, \quad \text{for all } \ t \geq T. \quad (4.4)$$

Define a continuous function $\Gamma(u)$ by setting

$$\Gamma(u)=\frac{u}{a(t)}+\sum_{j=2}^{m}\frac{\beta_j(t)}{a(t)}e^{u\tau_j^+}+[\frac{\beta_1(t)}{a(t)}-\frac{H(t)}{a(t)}]e^{u\tau_1^+},u\in[0,\ 1],\ t\in[T,\ +\infty). \tag{4.5}$$

Then, from (4.4), we have

$$\Gamma(0)=\sum_{j=2}^{m}\frac{\beta_j(t)}{a(t)}+[\frac{\beta_1(t)}{a(t)}-\frac{H(t)}{a(t)}]<\eta_0<1,\quad \text{for all}\ \ t\in[T,\ +\infty),$$

which implies that there exist two constants $\eta>0$ and $\lambda\in(0,1]$ such that

$$\Gamma(\lambda)=\frac{\lambda}{a(t)}+\sum_{j=2}^{m}\frac{\beta_j(t)}{a(t)}e^{\lambda\tau_j^+}+[\frac{\beta_1(t)}{a(t)}-\frac{H(t)}{a(t)}]e^{\lambda\tau_1^+}<\eta<1,\quad \text{for all}\ \ t\in[T,\ +\infty). \tag{4.6}$$

We consider the Lyapunov functional

$$V(t)=x(t)e^{\lambda t}. \tag{4.7}$$

Calculating the derivative of $V(t)$ along the solution $x(t)$ of (1.2), in view of (2.1) and (4.3), we have

$$\begin{aligned}
V'(t) &= -a(t)x(t)e^{\lambda t}+[\sum_{j=2}^{m}\beta_j(t)x(t-\tau_j(t))e^{-\gamma_j(t)x(t-\tau_j(t))}\\
&\quad +\beta_1(t)x(t-\tau_1(t))e^{-\gamma_1(t)x(t)}-H(t)x(t-\sigma(t))]e^{\lambda t}+\lambda x(t)e^{\lambda t}\\
&= (\lambda-a(t))x(t)e^{\lambda t}+\sum_{j=2}^{m}\beta_j(t)x(t-\tau_j(t))e^{\lambda t}e^{-\gamma_j(t)x(t-\tau_j(t))}\\
&\quad +[\beta_1(t)e^{-\gamma_1(t)x(t)}-H(t)]x(t-\tau_1(t))e^{\lambda t}\\
&\leq (\lambda-a(t))x(t)e^{\lambda t}+\sum_{j=2}^{m}\beta_j(t)x(t-\tau_j(t))e^{\lambda t}\\
&\quad +[\beta_1(t)-H(t)]x(t-\tau_1(t))e^{\lambda t},\ \text{for all}\ \ t\geq T.
\end{aligned} \tag{4.8}$$

Now, we claim that

$$V(t)=x(t)e^{\lambda t}<e^{\lambda T}(\max_{t\in[t_0-r,\ T]}x(t)+1):=M\quad \text{for all}\ \ t>T. \tag{4.9}$$

Contrarily, there must exist $t_*>T$ such that

$$V(t_*)=M\quad \text{and}\ \ V(t)<M\ \ \text{for all}\quad t<t_*, \tag{4.10}$$

which, together with (4.8) implies that

$$\begin{aligned}
0 &\leq V'(t_*) \\
&\leq (\lambda - a(t_*))x(t_*)e^{\lambda t_*} + \sum_{j=2}^{m} \beta_j(t_*)x(t_* - \tau_j(t_*))e^{\lambda t_*} \\
&\quad +[\beta_1(t_*) - H(t_*)]x(t_* - \tau_1(t_*))e^{\lambda t_*} \\
&= (\lambda - a(t_*))x(t_*)e^{\lambda t_*} + \sum_{j=2}^{m} \beta_j(t_*)x(t_* - \tau_j(t_*))e^{\lambda(t_* - \tau_j(t_*))}e^{\lambda \tau_j(t_*)} \\
&\quad +[\beta_1(t_*) - H(t_*)]x(t_* - \tau_1(t_*))e^{\lambda(t_* - \tau_1(t_*))}e^{\lambda \tau_1(t_*)} \\
&\leq \{(\lambda - a(t_*)) + \sum_{j=2}^{m} \beta_j(t_*)e^{\lambda \tau_j^+} + [\beta_1(t_*) - H(t_*)]e^{\lambda \tau_1^+}\}M. \qquad (4.11)
\end{aligned}$$

Thus,

$$0 \leq (\lambda - a(t_*)) + \sum_{j=2}^{m} \beta_j(t_*)e^{\lambda \tau_j^+} + [\beta_1(t_*) - H(t_*)]e^{\lambda \tau_1^+},$$

and

$$1 \leq \frac{\lambda}{a(t_*)} + \sum_{j=2}^{m} \frac{\beta_j(t_*)}{a(t_*)}e^{\lambda \tau_j^+} + [\frac{\beta_1(t_*)}{a(t_*)} - \frac{H(t_*)}{a(t_*)}]e^{\lambda \tau_1^+},$$

which contradicts with (4.6). Hence, (4.9) holds. It follows that

$$x(t) < Me^{-\lambda t} \text{ for all } t > T.$$

This completes the proof.

5. Examples and remarks

In this section, we present two examples to check the validity of our results we obtained in the previous sections.

Example 5.1. Consider the following Nicholson's blowflies model with a linear harvesting term:

$$\begin{aligned}
x'(t) &= -(1 + \frac{1}{1+t^2})x(t) + (10 + \cos^2 t)x(t - 2e^{|\arctan t|})e^{-x(t - 2e^{|\arctan t|})} \\
&\quad +(30 + \cos^4 t)x(t - e^{|\arctan t|})e^{-x(t)} - (20 + \cos^4 t)x(t - e^{|\arctan t|}). \qquad (5.1)
\end{aligned}$$

Then

$$a(t) = 1 + \frac{1}{1+t^2}, \ \beta_2(t) = 10 + \cos^2 t, \ \ \beta_1(t) = 30 + \cos^4 t, \ H(t) = 20 + \cos^4 t$$

$$\tau_2(t) = 2e^{|\arctan t|}, \quad \tau_1(t) = \sigma(t) = e^{|\arctan t|}, \quad r = 2e^{\frac{\pi}{2}}.$$

Thus

$$\inf_{t\in R}\{\beta_1(t) - H(t)\} = 10 > 0, \quad \text{and} \quad \tau_1(t) \equiv \sigma(t) \ \text{ for all } \ t \in R,$$

and

$$\liminf_{t\to+\infty}\left\{\frac{\beta_2(t)}{a(t)} + [\frac{\beta_1(t)}{a(t)} - \frac{H(t)}{a(t)}]\right\} > 10.$$

It follows that the Nicholson's blowflies model (5.1) satisfies all the conditions in Theorem 3.1. Hence, the model (5.1) is globally permanent on $C_+ = C([-2e^{\frac{\pi}{2}}, \ 0], (0, \ +\infty))$. This fact is verified by the numerical simulation in Figs. 1–2.

Example 5.2. Consider the following Nicholson's blowflies model with a linear harvesting term:

$$x'(t) = -(10 + \frac{1}{1+t^2})x(t) + (1+\cos^2 t)x(t - 2e^{|\arctan t|})e^{-x(t-2e^{|\arctan t|})}$$
$$+(3+\cos^4 t)x(t - e^{|\arctan t|})e^{-x(t)} - (2+\cos^4 t)x(t - e^{|\arctan t|}). \tag{5.2}$$

Then

$$a(t) = 10 + \frac{1}{1+t^2}, \ \beta_2(t) = 1+\cos^2 t, \quad \beta_1(t) = 3+\cos^4 t, \ H(t) = 2+\cos^4 t$$
$$\tau_2(t) = 2e^{|\arctan t|}, \quad \tau_1(t) = \sigma(t) = e^{|\arctan t|}, \quad r = 2e^{\frac{\pi}{2}}.$$

Thus, $\inf_{t\in R}\{\beta_1(t) - H(t)\} = 1 > 0$, and $\tau_1(t) \equiv \sigma(t)$ for all $t \in R$, and

$$\limsup_{t\to+\infty}\left\{\frac{\beta_2(t)}{a(t)} + [\frac{\beta_1(t)}{a(t)} - \frac{H(t)}{a(t)}]\right\} < \frac{2}{5}.$$

It follows that the Nicholson's blowflies model (5.2) satisfies all the conditions in Theorem 4.1, and the zero equilibrium point of the model (5.2) is globally exponentially stable on $C_+ = C([-2e^{\frac{\pi}{2}}, \ 0], (0, \ +\infty))$. Numerical simulations are given in Figs. 3–4.

Remark 5.1. To the best of our knowledge, few authors have considered the problems on the global dynamic behaviors of Nicholson's blowflies model with a linear harvesting term. It is clear that all the results in [2−9] and the references therein cannot be applicable to prove the global permanence of (5.1) and the global stability of (5.2). Moreover, in this present paper, we proposed a new approach to deal with the global dynamic behaviors for Nicholson's blowflies model with a linear harvesting term. Thus, the results of this present paper give a good reply to the open problem in [1] on the Nicholson's blowflies model with the linear harvesting term. Whether or not our results and method in this paper are available for studying the global stability on the periodic solutions or almost periodic solutions of Nicholson's blowflies model with a linear harvesting term, it is an interesting problem and we leave it as our work in the future.

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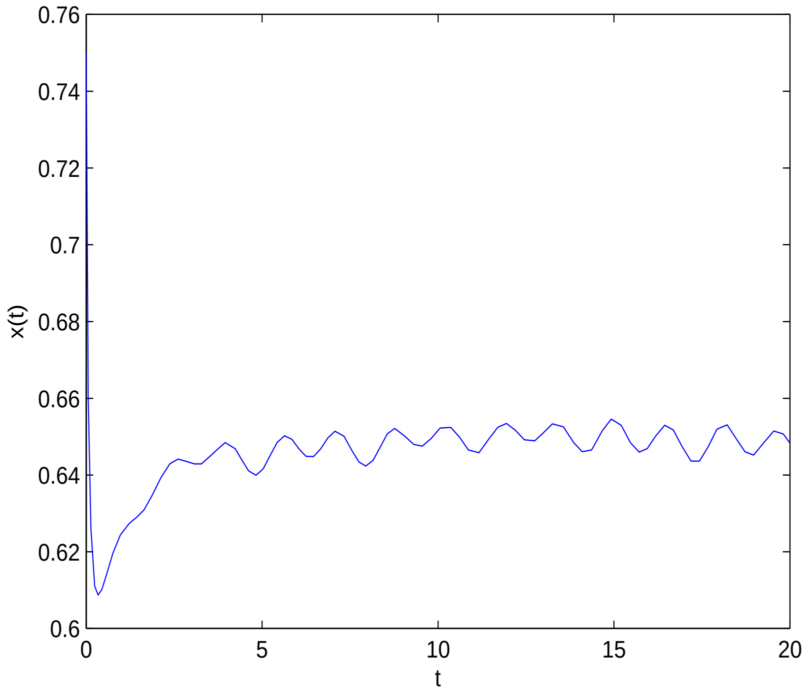

Fig. 1: Numerical solution $x(t)$ of equation (5.1) for initial value $\varphi(s) \equiv 0.75,\ s \in [-2e^{\frac{\pi}{2}},\ 0]$.

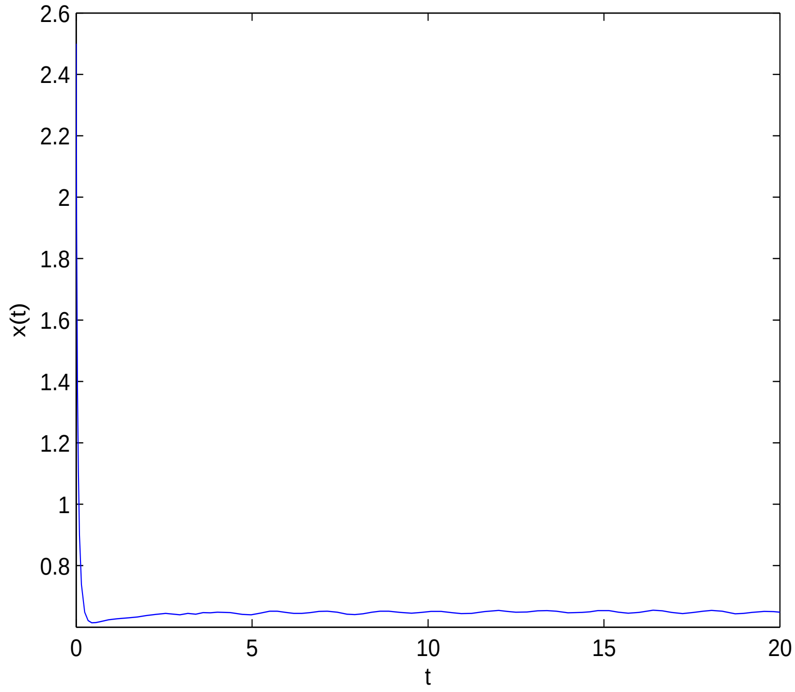

Fig. 2: Numerical solution $x(t)$ of equation (5.1) for initial value $\varphi(s) \equiv 2.5,\ s \in [-2e^{\frac{\pi}{2}},\ 0]$.

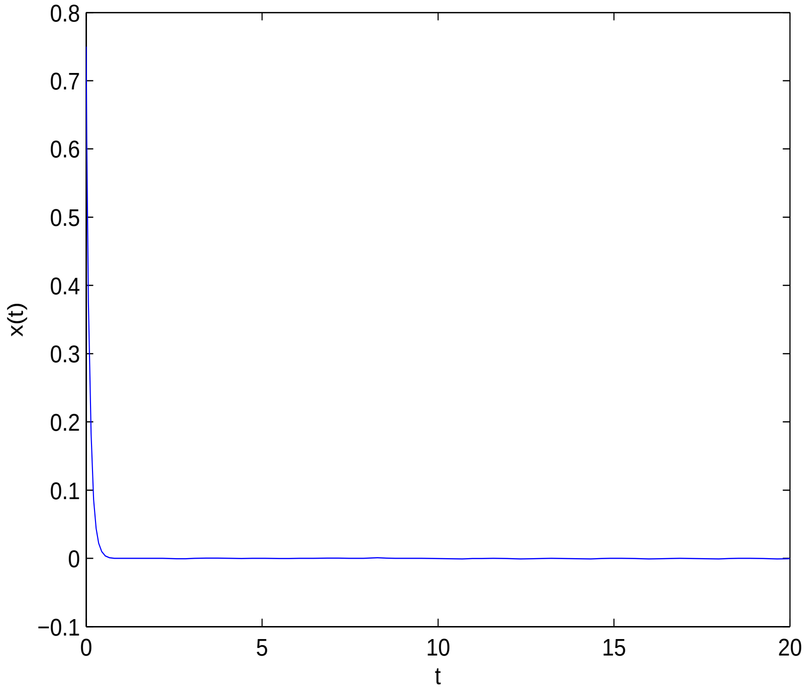

Fig. 3: Numerical solution $x(t)$ of equation (5.2) for initial value $\varphi(s) \equiv 0.75,\ s \in [-2e^{\frac{\pi}{2}},\ 0]$.

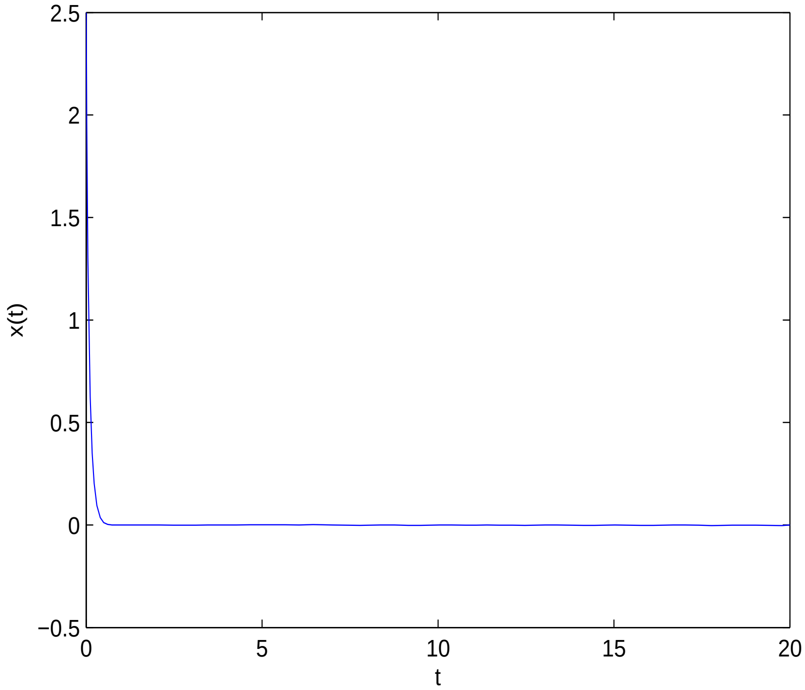

Fig. 4: Numerical solution $x(t)$ of equation (5.2) for initial value $\varphi(s) \equiv 2.5,\ s \in [-2e^{\frac{\pi}{2}},\ 0]$.