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Mean square exponential stability of stochastic delay cellular neural networks

Yingxin Guo[a,b]

[a]National Key Laboratory of Industrial Control Technology, Institute of Cyber-Systems and Control, Department of Control Engineering, Zhejiang University, Hangzhou 310027, China

[b]School of Mathematical Sciences, Qufu Normal University, Qufu, Shandong 273165, China

Abstract. By constructing suitable Lyapunov functionals and combining with matrix inequality technique, a new simple sufficient condition is presented for the exponential stability of stochastic cellular neural networks with discrete delays. The condition contains and improves some of the previous results in the earlier references. These sufficient conditions only including those governing parameters of SDCNNs can be easily checked by simple algebraic methods. Finally, one example is given to demonstrate that the proposed criteria are useful and effective.

Keywords: Delay differential equations; Lyapunov functionals; Matrix inequality; Exponential stability

MSC: 34K20; 34K13; 92B20

1 Introduction

The dynamical behaviors of stochastic neural networks have appeared as a novel subject of research and applications, such as optimization, control, and image processing(see [1-12]). Obviously, finding stability criteria for these neural networks becomes an attractive research problem of importance. Some well results have just appeared, for example, in [1-5], for stochastic delayed Hopfield neural networks and stochastic Cohen-Grossberg neural networks, the linear matrix inequality approach is utilized to establish the sufficient conditions on global stability for the neural networks. In particular, in [2], by using the method of variation parameter and stochastic analysis, the sufficient conditions are given to guarantee the exponential stability of an equilibrium solution. However, there are few results about stochastic effects to the stability property of cellular neural networks with delays in the literature today.

In this paper, exponential stability of equilibrium point of stochastic cellular neural networks with delays(SDCNNs) is investigated. Following [13], that activation functions require Lipschitz conditions and boundedness, by utilizing general Lyapunov function, stochastic analysis, Young inequality method and Poincare contraction theory are utilized to derive the conditions guaranteeing the existence of periodic solutions of SDCNNs and the stability of periodic solutions. Different from the LMI (linear matrix inequality) approach [13], [15] and variation parameter method, the Young inequality method is firstly developed to investigate the stability of SDCNN. These sufficient conditions improve and extend the early works in Refs. [18,19], and they include those governing parameters of SDCNNs, so they can be easily checked by simple algebraic methods, comparing with the results of [13-17]. Furthermore, one example is given to demonstrate the usefulness of the results in this paper.

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The organization of this paper is as follows. In Section 2, problem formulation and preliminaries are given. In Section 3, some new results are given to ascertain the exponential stability of the neural networks with time-varying delays based on Lyapunov method. Section 4 gives an example to illustrate the effectiveness of our results.

2 Preliminaries and lemmas

This paper, we are concerned with the model of continuous-time neural networks described by the following integro-differential systems:

$$\begin{aligned} x_i'(t) &= -d_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j(t))) \\ &\quad + \sum_{j=1}^{n} c_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(x_j(s))ds + J_i, \qquad i = 1, 2, \ldots, n, \end{aligned} \tag{1}$$

or equivalently

$$x'(t) = -Dx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + C\int_{-\infty}^{t} K(t-s) f(x(s))ds + J. \tag{2}$$

where n denotes the number of the neurons in the network, $x_i(t)$ is the state of the ith neuron at time t, $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in R^n$, $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T \in R^n$ denote the activation functions of the jth neuron at time t, $D = diag(d_1, d_2, \ldots, d_n) > 0$ is a positive diagonal matrix, $A = (a_{ij})_{n\times n}, B = (b_{ij})_{n\times n}$ and $C = (c_{ij})_{n\times n}$ are the feedback matrix and the delayed feedback matrix, respectively, $J = (J_1, J_2, \ldots, J_n)^T \in R^n$ be a constant external input vector, the kernels $k_j : [0, +\infty) \to [0, +\infty)$ are piece continuous functions with $\int_0^{+\infty} k_j(s)ds = 1, K(t-s) = [k_1(t-s), k_2(t-s), \ldots, k_n(t-s)]$, the time delay $\tau_j(t)$ is any nonnegative continuous function with $0 \le \tau_j(t) \le \tau$, where τ is a constant, $\tau(t) = [\tau_1(t), \tau_2(t), \ldots, \tau_n(t)]$.

In our analysis, we will employ that each $f_i, i = 1, 2, \ldots, n$ is bounded and satisfying the following condition:
(H)There exist constant scalars $L_i > 0$ such that

$$0 \le \frac{f_i(\eta_1) - f_i(\eta_2)}{\eta_1 - \eta_2} \le L_i, \qquad \forall\ \eta_1, \eta_2 \in R, \eta_1 \ne \eta_2.$$

This class of functions is clearly more general than both the usual sigmoid activation functions and the piecewise linear function:$f_i(x) = \frac{1}{2}(|x+1| - |x-1|)$, which is used in [11].

The initial conditions associated with system (1) are of the form

$$x_i(t) = \phi_i(t), \qquad t \in (-\infty, 0], \qquad i = 1, 2, \ldots, n,$$

in which $\phi_i(t)$ are continuous for $t \in (-\infty, 0]$.

Assume $x^*(t) = [x_1^*(t), x_2^*(t), \ldots, x_n^*(t)]^T$ is an equilibrium of Eq. (1), one can derive from (1) that the transformation $y_i = x_i - x_i^*$ transforms system (1) or (2) into the following system:

$$
\begin{aligned}
y_i'(t) = & -d_i y_i(t) + \sum_{j=1}^{n} a_{ij} g_j(y_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(y_j(t-\tau_j(t))) \\
& + \sum_{j=1}^{n} c_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(y_j(s)) ds, \qquad i = 1, 2, \ldots, n,
\end{aligned} \tag{3}
$$

where $g_j(y_j(t)) = f_j(y_j(t) + x_j^*) - f_j(x_j^*)$, or,

$$
y'(t) = -Dy(t) + Ag(y(t)) + Bg(y(t-\tau(t))) + C \int_{-\infty}^{t} K(t-s) g(y(s)) ds. \tag{4}
$$

Note that since each function $f_j(\cdot)$ satisfies the hypothesis (H), hence, each $g_j(\cdot)$ satisfies

$$
g_j^2(\eta_j) \le L_j^2 \eta_j^2, \forall\ \eta_j \in R,
$$

$$
\eta_j g_j(\eta_j) \ge \frac{g_j^2(\eta_j)}{L_j}, \forall\ \eta_j \in R,
$$

$$
g_j(0) = 0.
$$

To prove the stability of x^* of Eq. (1), it is sufficient to prove the stability of the trivial solution of Eq. (3) or (4).

Consider the following stochastic delayed recurrent neural networks with time varying delay

$$
\begin{cases}
dy_i(t) = \Big[-d_i y_i(t) + \sum_{j=1}^{n} a_{ij} g_j(y_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(y_j(t-\tau_j(t))) \\
\qquad + \sum_{j=1}^{n} c_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(y_j(s)) ds \Big] dt + \sum_{j=1}^{n} \sigma_{ij}(t, y_j(t), y_j(t-\tau_j(t))) dw_j(t) \\
y_i(t) = \phi_i(t),\ -\infty < t \le 0,\ \phi \in L^2_{\mathscr{F}_0}((-\infty, 0], R^n).
\end{cases} \tag{5}
$$

or equivalently

$$
\begin{cases}
dy(t) = \Big[-Dy(t) + Ag(y(t)) + Bg(y(t-\tau(t))) + C \int_{-\infty}^{t} K(t-s) g(y(s)) ds \Big] dt \\
\qquad + \sigma(t, y(t), y(t-\tau(t))) dw(t) \\
y(t) = \phi(t),\ -\infty < t \le 0,\ \phi \in L^2_{\mathscr{F}_0}((-\infty, 0], R^n).
\end{cases} \tag{6}
$$

where $i = 1, 2, \ldots, n; w(t) = (w_1(t), w_2(t), \ldots, w_n(t))^T$ is an n-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathscr{F}, P)$ with a natural filtration $\{\mathscr{F}_t\}_{t\ge 0}$ generated by $\{w(s) : 0 \le s \le t\}$, where we associate Ω with the canonical space generated by $w(t)$, and denote by $\mathscr{F}$ the associated σ-algebra generated by $w(t)$ with the probability measure P. $\{\phi_i(s), -\infty < s \le 0\}$ is $C((-\infty, 0]; R^n)$-valued function, for $i = 1, 2, \ldots, n$, which is $\mathscr{F}_0$-measurable R^n-valued random variables, where $C((-\infty, 0]; R^n)$ is the space of all continuous R^n-valued functions defined on $(-\infty, 0]$ with a norm $\|\phi\| = \sup\{|\phi(t)| : -\infty \le t \le 0\}$ and $|\cdot|$ is the Euclidean norm of a vector $x \in R^n$. $\sigma(t, x, y) = (\sigma_{ij}(t, x_j, y_j))_{n\times n}$, where $\sigma_{ij}(t, x_j, y_j) : R^+ \times R \times R \to R$ is locally Lipschitz continuous and satisfies the linear growth condition as well, $\sigma_{ij}(t, x_j^*(t), x_j^*(t-\tau_j(t))) = 0$.

Let $|y(t)|, \|y(t)\|$ denote the norms of the vector $y(t) = [y_1(t), y_2(t), \dots, y_n(t)]^T$, which are defined as

$$|y(t)| = [\sum_{i=1}^{n} |y_i(t)|^2]^{\frac{1}{2}}.$$

$$\|y(t)\| = \sup_{-\infty \leq s \leq 0} [\sum_{i=1}^{n} |y_i(t+s)|^2]^{\frac{1}{2}}.$$

Definition 1. The solution $y(t;\phi)$ of system (5) is said to be pth moment exponentially stable if there exists a pair of positive constants λ and c such that

$$\mathbb{E}\|y(t;\phi)\|^p \leq c\mathbb{E}\|\phi\|^p e^{-\lambda t},\ t \geq 0$$

holds for any ϕ, where $\mathbb{E}$ stands for the mathematical expectation operator. In this case

$$\lim_{t\to\infty} \sup \frac{1}{t} log(\mathbb{E}\|y(t;\phi)\|^p) \leq -\lambda. \tag{7}$$

The right-hand side of (7) is called the pth moment Lyapunov exponent of the solution. It is usually called the exponential stability in the mean square when $p = 2$.

Let $C^{2,1}(R^n \times R^+; R^+)$ denote the family of all non-negative functions $V(y,t)$ on $R^n \times R^+$ which are continuously twice differentiable in y and once differentiable in t. For each $V \in C^{2,1}(R^n \times R^+, R^+)$, define an operator $\mathbb{L}V$ associated with stochastic delayed neural networks (5) from $R^n \times R^+ \to R^+$ by

$$\begin{aligned}\mathbb{L}V(y(t),t) =& V_t(y,t) + V_y(y,t)\Big[-Dy(t) + Ag(y(t)) + Bg(y(t-\tau(t))) + C\int_{-\infty}^{t} K(t-s)g(y(s))ds\Big]dt \\ &+ \frac{1}{2} trace[\sigma^T(t, y(t), y(t-\tau(t)))V_{yy}(y,t)\sigma(t, y(t), y(t-\tau(t)))].\end{aligned} \tag{8}$$

where

$$V_t(y,t) = \frac{\partial V(y,t)}{\partial t}, V_y(y,t) = \Big(\frac{\partial V(y,t)}{\partial y_1}, \frac{\partial V(y,t)}{\partial y_2}, \dots, \frac{\partial V(y,t)}{\partial y_n}\Big), V_{yy}(y,t) = \Big(\frac{\partial^2 V(y,t)}{\partial y_i \partial y_j}\Big)_{n\times n}, \tag{9}$$

$i, j = 1, 2, \dots, n.$

In the following, we will use the notation $A > 0$ (or $A < 0$) to denote a symmetric and positive definite (or negative definite) matrix. The notation A^T and A^{-1} means the transpose of and the inverse of a square matrix A. If A, B are symmetric matrices, $A > B (A \geq B)$ means that $A - B$ is positive definite (positive semi-definite).

In order to obtain our result, we need the following lemma

Lemma 2([20]). For any vectors $a, b \in R^n$, the inequality

$$2a^T b \leq a^T X^{-1} a + b^T X b$$

holds for any matric $X > 0$.

3 Stability analysis

In this section, we present and prove our main results.

Theorem 1. Assume that there exist positive diagonal matrices $M = diag(m_1, m_2, \dots, m_n),\ M_0, M_1$ such that $trace[\sigma^T(t, y(t), y(t-\tau(t)))M\sigma(t, y(t), y(t-\tau(t)))] \leq y^T(t)M_0y(t) + y^T(t-\tau(t))M_1y(t-\tau(t))$, then the equilibrium point of system (5) is exponentially stable in the mean square if there exist a positive diagonal matrix $P = diag(p_1, p_2, \dots, p_n)$ such that

$$\begin{aligned}&-2MD + M + M_0 + LPL + LP\overline{G_1}L + \overline{G_2}(M_1 + LPL)+\\&MAP^{-1}A^TM + MBP^{-1}B^TM + MCP^{-1}C^TM < 0,\end{aligned}$$

where $L = diag(L_1, L_2, \dots, L_n), \overline{G_1} = diag(\int_0^\infty k_1(s)e^s ds, \int_0^\infty k_2(s)e^s ds, \dots, \int_0^\infty k_n(s)e^s ds), \overline{G_2} = diag(e^{\tau_1(h_1^{-1}(t))}, e^{\tau_2(h_2^{-1}(t))}, \dots, e^{\tau_n(h_n^{-1}(t))})$, where $h_i^{-1}(t)$ expresses the inverse function of $h_i(t) = t - \tau_i(t)$.

Proof. Since

$$\begin{aligned}&-2MD + M + M_0 + LPL + LP\overline{G_1}L + \overline{G_2}(M_1 + LPL)+\\&MAP^{-1}A^TM + MBP^{-1}B^TM + MCP^{-1}C^TM < 0.\end{aligned}$$

We can choose a small $\varepsilon > 0$ such that

$$\begin{aligned}&-2MD + \varepsilon M + M_0 + LPL + LPG_1L + G_2(M_1 + LPL) + MAP^{-1}A^TM+\\&MBP^{-1}B^TM + MCP^{-1}C^TM < 0.\end{aligned}$$

where

$$\begin{aligned}&G_1 = diag(\int_0^\infty k_1(s)e^{\varepsilon s}ds, \int_0^\infty k_2(s)e^{\varepsilon s}ds, \dots, \int_0^\infty k_n(s)e^{\varepsilon s}ds),\\&G_2 = diag(e^{\varepsilon\tau_1(h_1^{-1}(t))}, e^{\varepsilon\tau_2(h_2^{-1}(t))}, \dots, e^{\varepsilon\tau_n(h_n^{-1}(t))}).\end{aligned}$$

Consider the following positive definite Lyapunov function defined by:

$$\begin{aligned}V(y(t), t) = e^{\varepsilon t}y^T(t)My(t) + \sum_{j=1}^n (m_{1j} + L_j^2p_j)\int_{t-\tau_j(t)}^t y_j^2(s)e^{\varepsilon(s+\tau_j(h_j^{-1}(s)))}ds\\+\sum_{j=1}^n p_j\int_0^\infty k_j(s)e^{\varepsilon s}\int_{t-s}^t g_j^2(y_j(u))e^{\varepsilon u}duds.\end{aligned}$$

By Ito's formula, we calculate and estimate $\mathbb{L}V(y(t),t)$ along the trajectories of system (5) as follows:

$$\begin{aligned}
\mathbb{L}V(y(t),t) &= \varepsilon e^{\varepsilon t}y^T(t)My(t) \\
&\quad + 2e^{\varepsilon t}y^T(t)M\Big[-Dy(t)+Ag(y(t))+Bg(y(t-\tau(t)))+C\int_{-\infty}^{t}K(t-s)g(y(s))ds\Big] \\
&\quad + e^{\varepsilon t}trace[\sigma^T(t,y(t),y(t-\tau(t)))M\sigma(t,y(t),y(t-\tau(t)))] \\
&\quad + \sum_{j=1}^{n}(m_{1j}+L_j^2p_j)y_j^2(t)e^{\varepsilon(t+\tau_j(h_j^{-1}(t)))} - e^{\varepsilon t}\sum_{j=1}^{n}(m_{1j}+L_j^2p_j)y_j^2(t-\tau_j(t)) \\
&\quad + e^{\varepsilon t}\sum_{j=1}^{n}p_j\int_0^{\infty}k_j(s)e^{\varepsilon s}g_j^2(y_j(t))ds - e^{\varepsilon t}\sum_{j=1}^{n}p_j\int_0^{\infty}k_j(s)g_j^2(y_j(t-s))ds \\
&= e^{\varepsilon t}\Big\{\varepsilon y^T(t)My(t) - 2y^T(t)MDy(t) + 2y^T(t)MAg(y(t)) \\
&\quad + 2y^T(t)MBg(y(t-\tau(t))) + 2y^T(t)MC\int_{-\infty}^{t}K(t-s)g(y(s))ds \\
&\quad + trace[\sigma^T(t,y(t),y(t-\tau(t)))M\sigma(t,y(t),y(t-\tau(t)))] \\
&\quad + \sum_{j=1}^{n}(m_{1j}+L_j^2p_j)y_j^2(t)e^{\varepsilon(\tau_j(h_j^{-1}(t)))} - \sum_{j=1}^{n}(m_{1j}+L_j^2p_j)y_j^2(t-\tau_j(t)) \\
&\quad + \sum_{j=1}^{n}p_jg_j^2(y_j(t))\int_0^{\infty}k_j(s)e^{\varepsilon s}ds - \sum_{j=1}^{n}p_j\int_0^{\infty}k_j(s)ds\int_0^{\infty}k_j(s)g_j^2(y_j(t-s))ds\Big\} \\
&\le e^{\varepsilon t}\Big\{\varepsilon y^T(t)My(t) - 2y^T(t)MDy(t) + 2y^T(t)MAg(y(t)) \\
&\quad + 2y^T(t)MBg(y(t-\tau(t))) + 2y^T(t)MC\int_{-\infty}^{t}K(t-s)g(y(s))ds \\
&\quad + trace[\sigma^T(t,y(t),y(t-\tau(t)))M\sigma(t,y(t),y(t-\tau(t)))] \\
&\quad + e^{\varepsilon(\tau(h^{-1}(t)))}y^T(t)(M_1+LPL)y(t) - y^T(t-\tau(t))(M_1+LPL)y(t-\tau(t)) \\
&\quad + g^T(y(t))PG_1g(y(t)) - \sum_{j=1}^{n}p_j\Big(\int_0^{\infty}k_j(s)g_j(y_j(t-s))ds\Big)^2\Big\} \\
&= e^{\varepsilon t}\Big\{\varepsilon y^T(t)My(t) - 2y^T(t)MDy(t) + 2y^T(t)MAg(y(t)) \\
&\quad + 2y^T(t)MBg(y(t-\tau(t))) + 2y^T(t)MC\int_{-\infty}^{t}K(t-s)g(y(s))ds \\
&\quad + trace[\sigma^T(t,y(t),y(t-\tau(t)))M\sigma(t,y(t),y(t-\tau(t)))] \\
&\quad + e^{\varepsilon(\tau(h^{-1}(t)))}y^T(t)(M_1+LPL)y(t) - y^T(t-\tau(t))(M_1+LPL)y(t-\tau(t)) \\
&\quad + g^T(y(t))PG_1g(y(t)) - \Big(\int_{-\infty}^{t}K(t-s)g(y(s))ds\Big)^TP\Big(\int_{-\infty}^{t}K(t-s)g(y(s))ds\Big)\Big\},
\end{aligned} \tag{10}$$

From Lemma 2, we have

$$\begin{aligned}
2y^T(t)MAg(y(t)) &\le y^T(t)MAP^{-1}A^TM^Ty(t) + g^T(y(t))Pg(y(t)) \\
&\le y^T(t)(MAP^{-1}A^TM + LPL)y(t);
\end{aligned} \tag{11}$$

$$
\begin{aligned}
2y^T(t)MBg(y(t-\tau(t))) &\le y^T(t)MBP^{-1}B^TMy(t) + g^T(y(t-\tau(t)))Pg(y(t-\tau(t))) \\
&\le y^T(t)MBP^{-1}B^TMy(t) + y^T(t-\tau(t))LPLy(t-\tau(t));
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
2y^T(t)MC\int_{-\infty}^{t} K(t-s)g(y(s))ds &\le y^T(t)MCP^{-1}C^TMy(t) \\
&+ \Big(\int_{-\infty}^{t} K(t-s)g(y(s))ds\Big)^T P\Big(\int_{-\infty}^{t} K(t-s)g(y(s))ds\Big).
\end{aligned} \tag{13}
$$

From (10-13), we have

$$
\begin{aligned}
\mathbb{L}V(y(t),t) \le e^{\varepsilon t}y^T(t)[&-2MD + \varepsilon M + M_0 + LPL + LPG_1L + G_2(M_1 + LPL) + MAP^{-1}A^TM + \\
&MBP^{-1}B^TM + MCP^{-1}C^TM]y(t) \le 0.
\end{aligned}
$$

and so,

$$
\mathbb{E}V(y,t) \le \mathbb{E}V(y,0), \quad t > 0
$$

Since

$$
\begin{aligned}
\mathbb{E}V(y,0) &= \mathbb{E}\sum_{i=1}^{n}\Big[m_i|y_i(0)|^2 + (m_{1i} + L_i^2p_i)\int_{-\tau_i(0)}^{0} y_i^2(s)e^{\varepsilon(s+\tau_i(h_i^{-1}(s)))}ds \\
&\quad + p_i\int_0^{\infty} k_i(s)e^{\varepsilon s}\Big(\int_{-s}^{0} g_i^2(y_i(u))e^{\varepsilon u}du\Big)ds \\
&\le \mathbb{E}\sum_{i=1}^{n}\Big[m_i|y_i(0)|^2 + (m_{1i} + L_i^2p_i)\int_{-\tau_i(0)}^{0} |y_i(s)|^2 ds \\
&\quad + p_iL_i\int_0^{\infty} k_i(s)e^{\varepsilon s}\Big(\int_{-s}^{0} e^{\varepsilon u}du\Big)ds \sup_{-\infty\le u\le 0}|y_i(u)|^2\Big] \\
&\le \max_{1\le i\le n}\Big[m_i + \tau(m_{1i} + L_i^2p_i) + \frac{1}{\varepsilon}p_iL_i\int_0^{\infty} k_i(s)(e^{\varepsilon s}-1)ds\Big]\mathbb{E}\sum_{i=1}^{n}\sup_{-\infty\le u\le 0}|y_i(u)|^2
\end{aligned}
$$

where m_{1i} are entries of the matrix M_1 and

$$
\mathbb{E}V(y,t) \ge e^{\varepsilon t}\mathbb{E}\sum_{i=1}^{n} m_i|y_i(t)|^2 \ge e^{\varepsilon t}\min_{1\le i\le n} m_i\mathbb{E}\sum_{i=1}^{n}|y_i(t)|^2, \quad t > 0
$$

We easily obtain that

$$
\mathbb{E}\|y(t;\phi)\|^2 \le c\mathbb{E}\|\phi\|^2e^{-\varepsilon t},\ t \ge 0
$$

where $c \ge 1$ is a constant. The proof is complete.

When $C = 0$, the system (5) or (6) turns into following system:

$$
\begin{cases}
dy(t) = \Big[-Dy(t) + Ag(y(t)) + Bg(y(t-\tau(t)))\Big]dt + \sigma(t, y(t), y(t-\tau(t)))dw(t) \\
y(t) = \phi(t),\ -\infty \le t \le 0,\ \phi \in L^2_{\mathscr{F}_0}([-\infty, 0], R^n).
\end{cases} \tag{14}
$$

We can easily obtain the following corollary

Corollary 1. Assume that there exist positive diagonal matrices $M = diag(m_1, m_2, \ldots, m_n)$, M_0, M_1 such that $trace[\sigma^T(t, y(t), y(t-\tau(t)))M\sigma(t, y(t), y(t-\tau(t)))] \leq y^T(t)M_0y(t) + y^T(t-\tau(t))M_1y(t-\tau(t))$, then the equilibrium point of system (14) is exponentially stable in the mean square if there exist a positive diagonal matrix $P = diag(p_1, p_2, \ldots, p_n)$ such that

$$\begin{aligned}&-2MD + M + M_0 + LPL + LP\overline{G_1}L + \overline{G_2}(M_1 + LPL)+\\ &MAP^{-1}A^TM + MBP^{-1}B^TM < 0.\end{aligned}$$

When the feedback matrix $A = 0$ in Theorem 1, we can easily obtain the following corollary

Corollary 2. Assume that there exist positive diagonal matrices $M = diag(m_1, m_2, \ldots, m_n)$, M_0, M_1 such that $trace[\sigma^T(t, y(t), y(t-\tau(t)))M\sigma(t, y(t), y(t-\tau(t)))] \leq y^T(t)M_0y(t) + y^T(t-\tau(t))M_1y(t-\tau(t))$, then the equilibrium point of system (5) is exponentially stable in the mean square if there exist a positive diagonal matrix $P = diag(p_1, p_2, \ldots, p_n)$ such that

$$\begin{aligned}&-2MD + M + M_0 + LPL + LP\overline{G_1}L + \overline{G_2}(M_1 + LPL)+\\ &MBP^{-1}B^TM + MCP^{-1}C^TM < 0.\end{aligned}$$

When the delayed feedback matrix $C = 0$, the feedback matrix $A = 0$, in Theorem 1, we can easily obtain the following corollary

Corollary 3. Assume that there exist positive diagonal matrices $M = diag(m_1, m_2, \ldots, m_n)$, M_0, M_1 such that $trace[\sigma^T(t, y(t), y(t-\tau(t)))M\sigma(t, y(t), y(t-\tau(t)))] \leq y^T(t)M_0y(t) + y^T(t-\tau(t))M_1y(t-\tau(t))$, then the equilibrium point of system (5) is exponentially stable in the mean square if there exist a positive diagonal matrix $P = diag(p_1, p_2, \ldots, p_n)$ such that

$$-2MD + M + M_0 + LPL + LP\overline{G_1}L + \overline{G_2}(M_1 + LPL) + MBP^{-1}B^TM < 0.$$

Remark. Obviously, the results in Corollary 1,2,3 are more simple than Theorem 2 in [5] and Theorem 1 in [15,16]. Thus, Theorem 1 above generalizes the result in [5,15,16].

4 An example

In this section, an example is used to demonstrate that the method presented in this paper is effective.

Example. Consider the two state neural networks (5) or (6) with the following parameters:

$$A = (a_{ij})_{2\times2} = \begin{pmatrix} 0.1 & 0.7 \\ 0.3 & 0.1 \end{pmatrix}, \; B = (b_{ij})_{2\times2} = \begin{pmatrix} 0.2 & 0.3 \\ -0.3 & 0.2 \end{pmatrix},$$

$$C = (a_{ij})_{2\times2} = \begin{pmatrix} 0.3 & -0.1 \\ -0.1 & 0.7 \end{pmatrix}, \; D = (d_{ij})_{2\times2} = \begin{pmatrix} 3.1 & 0 \\ 0 & 3.0 \end{pmatrix},$$

$$\sigma_{11}(t, y_1(t), y_1(t-\tau_1(t))) = \frac{y_1(t)}{2} + \frac{y_1(t-\tau_1(t))}{2}, \; \sigma_{12}(t, y_2(t), y_2(t-\tau_2(t))) = 0,$$

$$\sigma_{21}(t, y_1(t), y_1(t-\tau_1(t))) = 0, \; \sigma_{22}(t, y_2(t), y_2(t-\tau_2(t))) = \frac{2y_2(t)}{5} + \frac{2y_2(t-\tau_2(t))}{5}$$

where $\tau_1(t) = \tau_2(t) = \frac{1}{4}e^{-4t} + \frac{1}{4}\sin t$, the activation function $f_1(t) = \cos\frac{t}{3} + \frac{t}{3}, f_2(t) = \sin\frac{t}{2} + \frac{t}{4}$, and the kernel $k_1(t) = k_2(t) = \frac{1}{5}e^{-5t}$. Clearly, $f_i(i = 1, 2)$ satisfies the hypothesis with $L_1 = L_2 = 1$ and $k_i(i = 1, 2)$ satisfies $\int_0^\infty k_i(s)ds = 1$. Let $h_i(t) = t - \tau_i(t) = t - \frac{1}{4}e^{-4t} - \frac{1}{4}\sin t(i = 1, 2)$, then $h_i'(t) = 1 + e^{-4t} - \frac{1}{4}\cos t > 0$. Hence the inverse function of $h_i(t)$ exists. Taking $M = I$, where I denotes the identity matrix of size n, and

$$M_0 = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.32 \end{pmatrix}, \ M_1 = \begin{pmatrix} 0.25 & 0 \\ 0 & 0.16 \end{pmatrix}.$$

Choose $P = I$, then we have

$$\begin{aligned} &-2MD + M + M_0 + LPL + LP\overline{G_1}L + \overline{G_2}(M_1 + LPL) + \\ &MAP^{-1}A^TM + MBP^{-1}B^TM + MCP^{-1}C^TM \leq \begin{pmatrix} -0.08 & 0 \\ 0 & -0.17 \end{pmatrix} < 0, \end{aligned}$$

Therefore, by theorem 1, the equilibrium point of Eq. (1) is exponentially stable in the mean square.

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