Abstract

An elementary geometric method is given to study nonlinear second order differential equations with step function coefficient

$$x'' + a^2(t)g(x) = 0, \qquad a(t) := a_k, \text{ if } t_{k-1} \le t < t_k \quad (k \in \mathbb{N})$$

where $a_k > 0$, $t_k \nearrow \infty$ as $k \to \infty$. The equation is rewritten into a discrete dynamical system on the plane. The method is applied to the excited pendulum equation when $g(x) = \sin x$. Starting from the usual periodic model, the problem of parametric resonance (problem of swinging) is treated. It has been pointed out that the realistic model of swinging is not a periodically excited system, instead swing is a self-oscillating system. Finally, the classical Oscillation Theorem is extended to the nonlinear periodic pendulum equation

$$\psi'' + a^2(t)\sin\psi = 0,$$
$$a(t) = \begin{cases} \sqrt{\dfrac{g}{l-\varepsilon}} & \text{if } 2kT \le t < (2k+1)T, \\ \sqrt{\dfrac{g}{l+\varepsilon}} & \text{if } (2k+1)T \le t < (2k+2)T, \quad (k \in \mathbb{Z}_+), \end{cases}$$

where g and l denote the constant of gravity and the length of the pendulum, respectively; $\varepsilon > 0$ is a parameter measuring the intensity of swinging.