Stability threshold for scalar linear periodic delay differential equations

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We investigate the scalar periodic delay-differential equation

$$\dot{x}(t) = -a(t)x(t) + b(t)x(t-1), \tag{1}$$

where a, b are assumed to be P-periodic continuous real functions with $a(t) \geq 0$ and $b(t) \geq 0$. In this talk, we prove that the stability threshold of (1) is $r = 0$, where

$$r := \int_0^P (b(s) - a(s))\,\mathrm{ds},$$

if $b(u+1) - a(u)$ does not change its sign. We also construct a class of equations of which r fails to be a stability threshold without the condition $b(u+1) - a(u)$ keeping its sign. Finding explicit stability threshold of (1) under weaker condition is an interesting open question.