2019 SZEGED WORKSHOP ON CONVEXITY
BOLYAI INSTITUTE, UNIVERSITY OF SZEGED, HUNGARY
APRIL 5–6, 2019

Schedule and abstracts

All talks will take place in the **Bolyai Lecture Hall** on the 2nd floor of the Bolyai Building of the University of Szeged at Aradi vértanúk tere 1., 6720 Szeged, Hungary.

Friday (April 5, 2019)

8:00–: Registration
8:40–8:45: *Opening*
8:45–9:35: Martin Henk: *On dual curvature measures*
9:40–10:00: Jesús Yepes Nicolás: *On discrete Borell-Brascamp-Lieb inequalities*
10:10–10:40: coffee break (Farkas lecture hall)
10:40–11:30: Andrea Colesanti: *Valuations on Lipschitz functions*
11:40–12:00: Nico Lombardi: *k-homogeneous valuations on quasi-concave functions*
12:10–14:00: lunch break
14:00–14:50: Martina Juhnke-Kubitzke: *Balanced shellings and moves on balanced manifolds*
15:00–15:20: Endre Makai Jr.: *Characterizations of balls*
15:30–15:50: Georg Hofstätter: *Blaschke-Santaló inequalities for Minkowski Endomorphisms*
16:00–16:30 coffee break (Farkas lecture hall)
16:30–16:50: Árpád Kurusa: *Quadrireciprocal points and Riemann points of a convex body*
16:55–17:15: Alexandru Kristály: *Equality in geometric inequalities via optimal mass transportation: how to get convexity?*
17:20–17:40: Angshuman Robin Goswami: *Decomposition of Phi-Convex functions*
18:30–21:00: Conference reception (Bolyai lecture hall)

Saturday (April 6, 2019)

8:00–8:50: Matthias Reitzner: *Random Boundary Polytopes*
9:00–9:20: Thomas Hack: *Randomized Urysohn-type inequalities*
9:30–9:50: Florian Besau: *Spherical Centroid Bodies*
10:00–10:30: coffee break (Farkas lecture hall)
10:30–11:20: Daniel Hug: *Splitting tessellations in spherical space*
11:30–11:50: Zsolt Lángi: *The isoperimetric ratio decreases monotonically in the Eikonal abrasion model*
12:00–14:00: lunch break
14:00–14:50: Gabriele Bianchi: *Convergence of symmetrization processes*
15:00–15:20: István Talata: *A generalization of the problem of six cylinders*
15:30–15:50: Antal Joós: *On covering the flat torus by congruent discs*
16:00–16:30: coffee break (Farkas lecture hall)
16:30–16:50: Gergely Ambrus: *Polarization, sign sequences and isotropic vector systems*
16:55–17:15: Tamás Zarnócz: *On the volume bound in the Dvoretzky-Rogers lemma*

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Polarization, sign sequences and isotropic vector systems

Gergely Ambrus
MTA Rényi Institute, Hungary

Let $\omega_n = \{u_1, \ldots, u_n\}$ be a system of n unit vectors in $\mathbb{R}^d$. For $p > 0$, we define the ℓ_p-potential function of ω_n by

$$U^p(\omega_n, v) = \sum_{i=1}^{n} |\langle u_i, v\rangle|^p,$$

and the ℓ_p*-polarization* of ω_n is defined by

$$M^p(\omega_n) = \max_{v \in S^{d-1}} U^p(\omega_n, v).$$

The n-th ℓ_p-polarization constant of S^{d-1} is given by

$$M_n^p(S^{d-1}) = \min_{\omega_n \subset S^{d-1}, \#\omega_n = n} M^p(\omega_n).$$

We determine the order of magnitude of $M_n^p(S^{d-1})$ for a fixed p, as $n, d \to \infty$. For $p = 2$, as well as for several values of p in the case $d = 2$, we are able to determine the exact value of the polarization constant of the sphere. For $p = 1$, the problem reduces to a question involving sign sequences, while for $p = 2$, the extremal vector systems turn out to be isotropic vectors systems.

A joint work with Sloan Nietert.

Spherical Centroid Bodies

Florian Besau
TU Wien, Austria

The centroid body of a centrally-symmetric convex body is a classical affine construction which was first considered by Blaschke in 3-dimensions and later by Petty in all dimensions. Petty was able to relate the volume of the centroid body with the expected volume of a random simplex inside the body. Hence, he was able to reinterpret Busemann's simplex inequality and established what is now known as the Busemann–Petty centroid inequality: the volume ratio of the centroid body is minimized for ellipsoids. Applying the classical Blaschke–Sanatló inequality on this inequality, one obtains the polar Busemann–Petty centroid inequality. With the generalization of the Brunn–Minkowski theory to the L_p and Orlicz setting new strong L_p and Orlicz extensions of these inequalities were obtained and numerous connections with asymptotic geometric analysis, geometric tomography, integral geometry and information theory were uncovered.

In recent joint work together with Thomas Hack, Peter Pivovarov and Franz E. Schuster we introduce spherical centroid bodies for centrally-symmetric (spherical) convex bodies on the Euclidean unit sphere in n-dimensions. We show that two natural definitions, one geometric and one probabilistic, lead to the same notion and establish various properties of the spherical centroid body. Most importantly we establish an inequality similar in nature to the polar Busemann–Petty centroid inequality. However, at this point a spherical analog for the Busemann–Petty centroid inequality remains an important open conjecture.

Convergence of symmetrization processes

Gabriele Bianchi
University of Florence, Italy

(joint research with R. J. Gardner and P. Gronchi)

It is known that there are sequences (H_m) of subspaces such that if K is a convex body and $\diamondsuit_H$ denotes Steiner or Minkowski symmetrization with respect to H then the sequence $\left(\diamondsuit_{H_m}\diamondsuit_{H_{m-1}}\ldots\diamondsuit_{H_1}K\right)$ converges to a ball.

- We study the same phenomenon for different symmetrizations $\diamondsuit_H$ and we also study the dependence on the symmetrization of the "rounding" sequence (H_m) of subspaces. We find conditions on a generic symmetrization $\diamondsuit_H$ that grant that any sequence of subspaces (H_m) which is "rounding" for Steiner or Minkowski symmetrizations is also "rounding" for $\diamondsuit_H$.

- Is it more difficult to "round" a compact set K than a convex set? We prove that, at least when dealing with Steiner, Schwartz or Minkowski symmetrizations, a sequence (H_m) of subspaces is "rounding" in the class of compact sets if and only if it is so in the class of convex bodies.
- A problem that is relevant in this research and on which we presents some results is the following: *Let $1 \leq i \leq n-1$, let $U_1, \ldots, U_k$ be i-dimensional subspaces of $\mathbb{R}^n$ and let $R_{U_i} \in O(n)$ denote reflection with respect to U_i. For which choices of $U_1, \ldots, U_k$ the closure of the subgroup of $O(n)$ generated by $R_{U_1}, \ldots, R_{U_k}$ acts transitively on S^{n-1}?*

Valuations on Lipschitz functions

Andrea Colesanti
University of Florence, Italy

The study of valuations on spaces of functions started quite recently, mainly motivated by the corresponding theory of valuations on convex bodies. The aim of the talk is to present a synthetic survey on the state of the art in this area, and to describe some recent results obtained in collaboration with Daniele Pagnini, Pedro Tradacete and Ingnacio Villanueva,, concerning valuations on the space of Lipschitz functions, verifying various types of invariance properties.

Decomposition of Phi-Convex functions

Angshuman Robin Goswami
University of Debrecen, Hungary

A real valued function f defined on a real open interval I is called Φ-convex if, for all $x, y \in I$, $t \in [0,1]$ it satisfies

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) + t\Phi\big((1-t)|x-y|\big) + (1-t)\Phi\big(t|x-y|\big),$$

where $\Phi : [0, \ell(I)[\to \mathbb{R}_+$, is a given non negative error function.

The main results of the paper offer various characterizations of Φ-convexity. One of the main result states that f is Φ-convex if and only if f can be decomposed into the sum of a convex function and a Φ-Hölder function

Randomized Urysohn-type inequalities

Thomas Hack
TU Wien, Austria

As a natural analog of Urysohn's inequality in Euclidean space, Gao, Hug, and Schneider showed in 2003 that in spherical or hyperbolic space, the probability that a given convex body K meets a randomly chosen totally geodesic hypersurface is minimized when K is a geodesic ball. We present a random extension of this result by taking K to be the convex hull of finitely many points drawn according to a probability distribution and by showing that the minimum is attained at uniform distributions on geodesic balls. As a corollary, we obtain a randomized Blaschke–Santaló inequality on the sphere. This is joint work with P. Pivovarov.

On dual curvature measures

Martin Henk
TU Berlin, Germany

Huang, Lutwak, Yang and Zhang introduced in 2016 the so called dual curvature measures of convex bodies which may be regarded as the counterparts to the classical area measures within the dual Brunn-Minkowski theory. The problem to characterize these geometric measures is known as the dual Brunn-Minkowski problem. In the talk we will present necessary subspace concentration conditions for these measures and we will also discuss possible extensions to the so called relative dual curvature measures.

(Based on joint works with Károly Böröczky Jr. and Hannes Pollehn.)

Blaschke-Santaló inequalities for Minkowski Endomorphisms

Georg Hofstätter
TU Wien, Austria

In this talk, we present a family of new isoperimetric inequalities for monotone Minkowski endomorphisms that interpolates between the Blaschke-Santaló and the Urysohn inequality. Among this large family of inequalities, the only affine invariant inequality turns out to be the strongest one. An extension of the family to merely weakly monotone Minkowski endomorphisms is shown to be impossible.
(joint work with F.E. Schuster)

Splitting tessellations in spherical space

Daniel Hug
Karlsruhe Institute of Technology, Germany

We introduce and explore the concept of splitting tessellations and splitting tessellation processes in spherical space, which provide an alternative to the common Voronoi and great subsphere tessellations. Expectations, variances and covariances of spherical curvature measures (and hence of spherical or conical intrinsic volumes) induced by a splitting tessellation will be studied with the help of tools from spherical integral geometry and the valuation property. Also the spherical pair-correlation function of the (d-1)-dimensional Hausdorff measure will be computed explicitly and compared to its analogue for Poisson great hypersphere tessellations. If time permits, we shall also explain how the typical cell distribution and the distribution of the typical spherical maximal face of any dimension can be expressed as mixtures of the related distributions of Poisson great hypersphere tessellations.
(joint work with Christoph Thäle)

On covering the flat torus by congruent discs

Antal Joós
University of Dunaújváros, Hungary

We will consider coverings of the square flat torus by congruent discs of minimal radius. These are periodic disc coverings of the Euclidean plane by congruent discs of minimal radius. Let $r(k)$ be the greatest lower bound of the radii of k congruent discs such that the square flat torus can be covered by these discs. It will be proved that $r(3) = 5\sqrt{2}/18$ and $r(4) \leq 5/16$.

Balanced shellings and moves on balanced manifolds

Martina Juhnke-Kubitzke
University of Osnabrück, Germany

A classical result by Pachner states that two d-dimensional combinatorial manifolds with boundary are PL homeomorphic if and only they can be connected by a sequence of shellings and inverse shellings. We prove that for balanced, i.e., properly (d+1)-colored, manifolds such a sequence can be chosen such that balancedness is preserved in each step. As a key ingredient we establish that any two balanced PL homeomorphic combinatorial manifolds with the same boundary are connected by a sequence of basic cross-flips, as was shown recently by Izmestiev, Klee and Novik for balanced manifolds without boundary. Moreover, we enumerate combinatorially different basic cross-flips and show that roughly half of these suffice to relate any two PL homeomorphic manifolds. This is joint work with Lorenzo Venturello.

Equality in geometric inequalities via optimal mass transportation: how to get convexity?

Alexandru Kristály
Babes-Bolyai University, Romania

By using optimal mass transportation and a quantitative Hölder inequality, we provide estimates for the Borell-Brascamp-Lieb deficit on complete Riemannian manifolds. Accordingly, equality cases in Borell-Brascamp-Lieb inequalities (including Brunn-Minkowski and Prékopa-Leindler inequalities) are characterized in terms of the optimal transport map between suitable marginal probability measures. These results provide several qualitative applications both in the flat and non-flat frameworks. In particular, by using Caffarelli's regularity result for the Monge-Ampère equation (by using the convexity of the support of the target measure), we give a new proof of Dubuc's characterization of the equality in Borell-Brascamp-Lieb inequalities in the Euclidean setting. Moreover, precise characterization is provided for the equality in the Lott-Sturm-Villani-type distorted Brunn-Minkowski inequality on Riemannian manifolds in terms of geodesic convexity. Talk based on the paper by Z.M. Balogh and A. Kristály, Equality in Borell-Brascamp-Lieb inequalities on curved spaces. Adv. Math. 339 (2018), 453494.

Quadrireciprocal points and Riemann points of a convex body

Árpád Kurusa
University of Szeged, Hungary

At a point P inside a convex body $\mathcal{B}$ the i-chord function $f_{i;P}$ is defined for every unit vector $\mathbf{v}$ such that if P divides into two segments the chord in which $\mathcal{B}$ intersects the line ℓ of direction $\mathbf{v}$ passing through P, then $f_{i;P}(\mathbf{v})$ is the i-power sum of the lengths of the segments.

A point P is called quadrireciprocal if at P the (-1)-chord function is quadratic.

A point P of a convex body $\mathcal{B}$ is called Riemannian if at P the infinitesimal sphere of the Hilbert metric of $\mathcal{B}$ is quadratic.

In fact a point of a convex body is quadrireciprocal if and only if it is also Riemannian.

We prove that a twice differentiable convex body is an ellipsoid if and only if it has two quadrireciprocal points. This can also be phrased such that if a Hilbert geometry of twice differentiable boundary has two Riemannian points, then it is the hypebolic geometry.

The isoperimetric ratio decreases monotonically in the Eikonal abrasion model

Zsolt Lángi
Morphodynamics Research Group of the Hungarian Academy of Sciences, and Budapest University of Technology, Hungary

A few years ago the strange elongated shape of the interstellar asteroid 'Oumuamua raised a significant interest both in the scientific and public community. A possible explanation for this phenomenon was given by Domokos et al., who, using their already existing model for the abrasion of asteroids, pointed out that in this model shapes become more and more aspherical. To establish the mathematical background of this statement and motivated by a question of Lovász, in this talk we verify this statement from the mathematical point of view, and show that the isoperimetric ratio decreases in the Eikonal abrasion model. The proof relies entirely on tools from convex geometry.

k-homogeneous valuations on quasi-concave functions

Nico Lombardi
University of Florence, Italy

We are going to introduce valuations defined on quasi-concave functions that are continuous, w.r.t. hypo-convergence, translation invariant and k-homogeneous, with $0 \leq k \leq n$.

Characterizations of balls

ENDRE MAKAI JR.
MTA Rényi Insitute, Hungary

Let X be either S^d (spherical d-space), or $\mathbb{R}^d$ or H^d (hyperbolic d-space). Let $K, L \subset X$ be C^2_+ closed convex sets, with interiors non-empty. Let φ, ψ be congruences of X. A ball (sphere) in S^d is meant as one with radius less than $\pi/2$.

Theorem. All intersections $(\varphi K) \cap (\psi L)$ are centrally symmetric if and only if K, L are congruent balls.

As variants of this theorem, we can characterize those pairs $K.L$ for which only all compact above intersections are centrally symmetric, or only all small above intersections are centrally symmetric.

Theorem. Additionally suppose that $K.L$ have at each boundary points supporting spheres. Then all closed convex hulls $\mathrm{cl\,conv}\,[(\varphi K) \cup (\psi L)]$ are centrally symmetric if and only if K, L are congruent balls.

(Joint results with J. Jerónimo-Castro)

Random Boundary Polytopes

MATTHIAS REITZNER
University of Osnabrück, Germany

Choose N random points on the boundary of a convex body. The convex hull of these points generates a random polytope. We report on investigations concerning properties of such a random polytope if the convex set is either a smooth convex body or a simple polytope. Asymptotic formulas for the number of vertices and faces, and for the intrinsic volumes are our main interest.

In particular, if the random points are chosen on the boundary of a polytope, the investigations lead to random polytopes which are neither simple nor simplicial. The results contrast existing results when points are chosen in the interior of a convex set.

A generalization of the problem of six cylinders

ISTVÁN TALATA
Ybl College of Szent Isván University, Hungary

W. Kuperberg proposed the following problem in 1990: What is the maximum number of unit radius, infinitely long cylinders with mutually disjoint interiors that can touch a unit ball? Kuperberg conjectured that the answer is 6, but this problem is still unsolved. Now we consider a generalization of this problem for different metrics, when the unit ball is not a Euclidean ball, and the base of every cylinder is a translate of the orthogonal projection of the unit ball with respect to the axis of the cylinder. We show that at most 8 cylinders can be arranged in the above specified way when the unit ball is a 3-dimensional unit cube.

On discrete Borell-Brascamp-Lieb inequalities

JESÚS YEPES NICOLÁS
University of Murcia, Spain

If $f, g, h : \mathbb{R}^n \longrightarrow \mathbb{R}_{\geq 0}$ are non-negative measurable functions such that $h(x+y)$ is greater than or equal to the p-sum of $f(x)$ and $g(y)$, where $-1/n \leq p \leq \infty$, $p \neq 0$, then the classical Borell-Brascamp-Lieb inequality asserts that the integral of h is not smaller than the q-sum of the integrals of f and g, for $q = p/(np+1)$.

In this talk we will show a discrete analog of the above-mentioned result for the sum over finite subsets of the integer lattice $\mathbb{Z}^n$: under the same assumption as before, for $A, B \subset \mathbb{Z}^n$, then $\sum_{A+B} h \geq [(\sum_{\mathrm{r}_f(A)} f)^q + (\sum_B g)^q]^{1/q}$, where $\mathrm{r}_f(A)$ is obtained by removing points from A in a particular way, and depending on f. In particular, different Brunn-Minkowski type inequalities are obtained when considering certain discrete measures on $\mathbb{R}^n$.

We will also show that the classical Borell-Brascamp-Lieb inequality for Riemann integrable functions can be derived as a consequence of this new discrete version.

This is joint work with: David Iglesias López (University of Murcia).

On the volume bound in the Dvoretzky-Rogers lemma.

Tamás Zarnócz
University of Szeged, Hungary

The classical Dvoretzky-Rogers lemma provides a lower bound on the maximal volume of parallelotopes spanned by d vectors from a John decomposition of the identity in d-dimensional space. This lower bound was improved by Pelczyński and Szarek. Using the expectation of the squared volume of parallelotopes spanned by d independent random vectors distributed according to discrete isotropic measures we provide a probabilistic approach to the volumetric estimate in the Dvoretzky-Rogers lemma.

This talk contains joint results with Ferenc Fodor (University of Szeged, Hungary) and Márton Naszódi (Eötvös University, Hungary).