2015 SZEGED WORKSHOP ON CONVEXITY

BOLYAI INSTITUTE, UNIVERSITY OF SZEGED, HUNGARY

NOVEMBER 13–14, 2015

Schedule and abstracts

All talks will take place in the **Bolyai Lecture Hall** on the 2nd floor of the Bolyai Building of the University of Szeged at Aradi vértanúk tere 1., 6720 Szeged, Hungary.

Friday (November 13, 2015)

9:40–9:45 *Opening*

9:45–10:40 Peter M. Gruber: *On the Border between Convex Geometry and the Geometry of Numbers*

10:45–11:10 Sören Berg: *Lattice points in centered convex bodies*

11:10–11:35 coffee break

11:35–12:30 Károly J. Böröczky: *Isotropic measures, the Geometric Brascamp–Lieb inequality and a stronger form of the reverse isoperimetric inequality*

12:35–13:00 Susanna Dann: *Bounding marginal densities via affine isoperimetry*

13:00–14:30: lunch break

14:30–15:25: Imre Bárány: *Magic mirrors, universal points, Baire category*

15:30–15:55: György Gehér: *Can we determine a point lying in a simplex by its distances from the vertices?*

16:00–16:25: Csaba Vincze: *On Brickell's theorem*

16:25–16:50: coffee break

16:50–17:15: Zsolt Lángi: *A genealogy of convex solids via local and global bifurcations of gradient vector fields*

17:20–17:45: Tamás Zarnócz: *Covering the sphere by equal zones and related questions*

17:50–18:15: Károly Böröczky: *About densest packings of unit balls in the Euclidean 3-space*

Saturday (November 14, 2015)

9:45–10:40 Franz Schuster: *Log-concavity properties of Minkowski valuations*

10:45–11:10 Dan Ma: *Real-valued valuations on Sobolev spaces*

11:10–11:50 coffee break

11:50–12:15 Astrid Berg: *The Lutwak-Petty projection inequalities for Minkowski valuations*

12:20–12:45 Felix Dorrek: *Projection Functions, Area Measures and the Alesker-Fourier Transform*

12:50–13:15 Árpád Kurusa: *It pays to measure twice*

Magic mirrors, universal points, Baire category

IMRE BÁRÁNY
MTA Rényi Insitute, Hungary, and University College London, U.K.

An old result of Zamfirescu says that 'most mirrors are magic', meaning that for most convex bodies K in $\mathbb{R}^2$ most points of the plane lie on infinitely many normals of K. Most is meant both times in Baire category sense. We show (joint work with Laczkovich) that 'most mirrors are even more magic', namely, that in the above statement 'infinitely' can be replaced by 'continuum many'. In a different direction but also in Baire category sense we prove (joint work with Schneider) that most boundary points z of most convex bodies K in $\mathbb{R}^{\nVdash}$ are universal. The meaning of universal will be defined in the lecture.

The Lutwak-Petty projection inequalities for Minkowski valuations

ASTRID BERG
Vienna University of Technology, Austria

The Petty projection inequality is an affine isoperimetric inequality for the volume of the polar projection body of order $n-1$, which was generalized by Lutwak to projection bodies of order i. It was recently proven by Haberl and Schuster, that the Petty projection inequality can be extended to $(n-1)$-homogeneous Minkowski valuations intertwining rigid motions which are generated by zonoids of revolution.

In this talk we present a generalization of the Lutwak-Petty projection inequalities to such Minkowski valuations which are i-homogeneous. To this end we present a version of the Busemann-Petty centroid inequality for a generalized centroid body operator. We identify Lutwak's inequalities as the strongest among our family and relate our results to the classical isoperimetric inequalities, comparing volume and intrinsic volumes of a convex body and to a conjecture of Lutwak on affine quermassintegrals.

Lattice points in centered convex bodies

SÖREN BERG
Technische Universität Berlin, Germany

In this talk we will discuss upper bounds on the number of lattice points in a convex body having its centroid at the origin. We present an upper bound for arbitrary convex bodies and obtain a sharp upper bound for simplices in particular. These results are continuations of a theorem due to Hermann Minkowski dealing with the (stronger) centrally symmetric assumption. Furthermore, a relation to Ehrharts volume conjecture will be briefly discussed.

This is joint work with Martin Henk

About densest packings of unit balls in the Euclidean 3-space

KÁROLY BÖRÖCZKY
Eötvös University, Hungary

It is well known that there exist continuum many packings of unit balls in the Euclidean 3-space where each ball is touched by 12 others. In these packings, the Dirichlet-Voronoi cells are either rhombic dodecahedra or trapezo-rhombic dodecahedra, and the density of a unit ball with respect to its Dirichlet-Voronoi cell is $\pi/\sqrt{18}$ for both types of cells. For long, it has been widely believed that these arrangements are densest among all packings of unit balls. Most probably, some vague form of the conjecture goes back not only to Kepler but to Archimedes.

T. Hales' monumental work is a strong indication that the conjecture might hold. We present some argument in the talk why the conjecture is so difficult, and why one needs additional deep ideas to verify it.

Isotropic measures, the Geometric Brascamp–Lieb inequality and a stronger form of the reverse isoperimetric inequality

Károly J. Böröczky
MTA Rényi Institute of Mathematics, Hungary

Joint work with Ferenc Fodor and Daniel Hug.

After K. Ball found the "Geometric" form of the Brascamp-Lieb inequality, and applied this to verify the reverse isoperimetric inequality, his method proved to be a powerful tool in convexity. Later F. Barthe even found a reverse form of the Brascamp-Lieb inequality. The talk reviews the Geometric Brascamp-Lieb inequality and its reverse form, explains their proof based on optimal transportation due to F. Barthe, and indicates how these inequalities could be strengthened in some geometric significant special cases. Applications include the stability versions of the reverse isoperimetric inequality, and some volume inequalities for L_p zonoids due to K. Ball, F. Barthe, E. Lutwak, D. Yang and G. Zhang.

Bounding marginal densities via affine isoperimetry

Susanna Dann
Vienna University of Technology, Austria

Let μ be a probability measure on $\mathbb{R}^n$ with a bounded density f. We prove that the marginals of f on most subspaces are well-bounded. For product measures, studied recently by Rudelson and Vershynin, our results show there is a trade-off between the strength of such bounds and the probability with which they hold. Our proof rests on new affinely-invariant extremal inequalities for certain averages of f on the Grassmannian and affine Grassmannian. These are motivated by Lutwak's dual affine quermassintegrals for convex sets. We show that key invariance properties of the latter, due to Grinberg, extend to families of functions. The inequalities we obtain can be viewed as functional analogues of results due to Busemann–Straus, Grinberg and Schneider. As an application, we show that without any additional assumptions on μ, any marginal $\pi_E(\mu)$, or a small perturbation thereof, satisfies a nearly optimal small-ball probability.

This is joint work with Grigoris Paouris and Peter Pivovarov.

Projection Functions, Area Measures and the Alesker-Fourier Transform

Felix Dorrek
Vienna University of Technology, Austria

Joint work with Franz Schuster.

Dual to Koldobsky's notion of j-intersection bodies, the class of j-projection bodies is introduced, generalizing Minkowski's notion of projection bodies of convex bodies. We call a convex body K the j-projection body of another convex body L if

$$vol_j(K|E^{\perp}) = vol_{n-j}(L|E),$$

for all $(n-j)$-dimensional subspaces E.

A fundamental Fourier-analytic characterization of j-intersection bodies due to Koldobsky initiated further investigations of this class. Here a dual version of this theorem for j-projection bodies will be discussed that characterizes those bodies in terms of their area measures. It turns out that this characterization is closely related to another - valuation-theoretic - characterization involving the Alesker-Fourier transform.

Can we determine a point lying in a simplex by its distances from the vertices?

György Gehér
University of Szeged, Hungary

It is an elementary fact that if we fix an arbitrary set of $d+1$ affine independent points $\{p_0, \dots p_d\}$ in $\mathbb{R}^d$, then the Euclidean distances $\{|x-p_j|\}_{j=0}^d$ determine the point x in $\mathbb{R}^d$ (and therefore in the simplex $\mathrm{Conv}(p_0, \dots p_d)$) uniquely. In my talk I would like to investigate a similar problem in general normed spaces. Namely, I will present a characterization of those, at least d-dimensional, real normed spaces $(X, \|\cdot\|)$ for which every set of $d+1$ affine independent points $\{p_0, \dots p_d\} \subset X$, the distances $\{\|x-p_j\|\}_{j=0}^d$ determine the point x lying in the simplex $\mathrm{Conv}(p_0, \dots p_d)$ uniquely. Surprisingly, the characterization depends on d.

On the Border between Convex Geometry and the Geometry of Numbers

Peter M. Gruber
Vienna University of Technology, Austria

(i) From the normal bundle cone of a convex body one can read off easily important properties of the body, for example, whether the convex body has a tangent linear vectorfield, or what are its planar shadow boundaries.

(ii) The famous criterion of Voronoi says that a lattice packing of balls has locally maximum density if and only if it is perfect and eutactic. We extend it to lattice packings of smooth convex bodies with refined maximum properties of the density, using generalizations of the notions of perfection and eutaxy. The extensions yield lower bounds for the kissing numbers of dense lattice packings.

(iiii) Using the notion of well-rounded lattices, it is possible to prove an inequality between the packing and the covering radius of the lattice. This "trans which is of interest in context of the well known conjecture of Minkowski on the product of non-homogeneous linear forms. The normal bundle cone is used to give transparent definitions of the Voronoi type notions of perfection, eutaxy and semi-eutaxy and each of the latter properties implies well-roundedness. The mentioned results thus seem to be connected, the character of the connection still waiting for clarification.

Stochastic geometry in spherical space

Daniel Hug
Karlsruhe Institute of Technology (KIT), Germany

Stochastic geometry in Euclidean space $\mathbb{R}^d$ has been studied extensively and much progress has been made within the last 20 years. In many situations, the Euclidean setting is particularly convenient and fruitful, since the geometry is reasonably easy to visualize, Euclidean functionals such as the intrinsic volumes (Minkowski functionals) are thoroughly understood and the Euclidean isometry group has a simple (semidirect) product structure splitting into a translational and a rotational part. In particular, the translation group is often very useful for analyzing models in Euclidean stochastic geometry.

Very recently stochastic geometry in *spherical space* has come into focus. We start this talk with a study of random spherical polytopes generated as the spherically convex hull of random points sampled in a hemisphere. In contrast to the Euclidean case, we obtain closed form expressions (and asymptotic results) for some of the geometric characteristics of spherical polytopes (see [1]). In particular, we use the opportunity to introduce some of the relevant geometric functionals (intrinsic volumes, quermassintegrals, number of k-faces, k-face contents) in spherical space.

Second, we consider random tessellations of the unit sphere $\mathbb{S}^{d-1}$ generated by great subspheres of codimension 1 (the intersections of $\mathbb{S}^{d-1}$ with $(d-1)$-dimensional linear subspaces). Equivalently, we study conical tessellations of $\mathbb{R}^d$. For various geometric functionals we obtain mean value formulas for certain random cones, the Schläfli-cone S_n and the Cover-Efron cone C_n, which are dual to each other. The Schläfli cone is obtained by picking at random (with equal chances) one of the (Schläfli) cones generated by independent random linear subspaces $H_1, \dots, H_n \in G(d, d-1)$, which all follow the same distribution. In addition to mean value formulas, we also derive some explicit second order moments (and thus covariances) in the isotropic case (see [5]).

In Euclidean space, the problem of determining the asymptotic or limit shape (if it exists) of large cells in Poisson driven random tessellations has become known as Kendall's problem (see [2, 3, 4]). In spherical space, the statement of the problem has to be modified since "large cells" cannot occur. We indicate some recent progress on this problem, which usually leads to interesting isoperimetric problems (see [6]).

Finally, we shall provide the setting for studying Boolean models in spherical space and present Miles-type formulas (see [7]). In this context it is interesting to note that an immediate spherical analogue of Hadwiger's famous characterization theorem for Minkowski functionals in Euclidean space is unknown in the spherical setting. Hence, rotational integral-geometric formulas for functionals on the sphere have to be established in a different way.

This talk is based on joint work with I. Bárány, G. Last, A. Reichenbacher, M. Reitzner, R. Schneider, W. Weil.

References

[1] I. Bárány, D. Hug, M. Reitzner, R. Schneider. Random points in halfspheres. arXiv:1505.04672. Random Structures Algorithms (to appear)

[2] D. Hug, M. Reitzner, R. Schneider. The limit shape of the zero cell in a stationary Poisson hyperplane tessellation. Ann. Probab. 32 (2004), 1140-1167.

[3] D. Hug, R. Schneider. Asymptotic shapes of large cells in random tessellations. Geom. Funct. Anal. 17 (2007), 156–191.

[4] D. Hug, R. Schneider. Faces in Poisson-Voronoi mosaics. Probab. Theory and Relat. Fields. 151 (2011), 125-151.

[5] D. Hug, R. Schneider. Random conical tessellations. arXiv:1508.07768.

[6] D. Hug, A. Reichenbacher. Kendall's problem in spherical space. (in preparation)

[7] D. Hug, G. Last, W. Weil. Boolean models in spherical space. (in preparation)

It pays to measure twice

Árpád Kurusa
University of Szeged, Hungary

There are several situations that are similar to the following pair of statements about isoptics (the sets of points where a convex set subtends constant angle).

> There are non-spherical convex bodies that have spherical isoptics.
> If a convex body has two different spherical isoptics, then it is spherical.

These pairs demonstrate that *it pays to measure twice*.

In this talk we show several results where it pays to measure twice, and we prove an easy lemma (*of double measuring*) that can be used for the proof of every such result. This lemma establishes coincidence of domains based on an inequality of their two different measurements.

This is a joint work with Tibor Ódor (University of Szeged).

References

[1] Á. Kurusa and T. Ódor, Characterizations of balls by sections and caps, *Beitr. Alg. Geom.,* **56:2** (2015), 459–471;

[2] Á. Kurusa and T. Ódor, Spherical floating body, *Acta Sci. Math. (Szeged),* **81:3-4** (2015), accepted;

[3] Á. Kurusa and T. Ódor, Isoptic characterization of spheres, *J. Geom.,* **106:1** (2015), 63–73;

A genealogy of convex solids via local and global bifurcations of gradient vector fields

Zsolt Lángi
Budapest University of Technology and Economics, Hungary

Three-dimensional convex bodies can be classified in terms of the number and stability types of critical points on which they can balance at rest on a horizontal plane. For typical bodies these are nondegenerate maxima, minima, and saddle-points, the numbers of which provide a primary classification. Secondary and tertiary classifications use graphs to describe orbits connecting these critical points in the gradient vector field associated with each body. In previous work it was shown that these classifications are complete in that no class is empty. Here we construct 1- and 2-parameter families of convex bodies connecting members of adjacent primary and secondary classes and show that transitions between them can be realized by codimension 1 saddle-node and saddle-saddle (heteroclinic) bifurcations in the gradient vector fields. Our results indicate that all combinatorially possible transitions can be realized in physical shape evolution processes, e.g. by abrasion of sedimentary particles. The project is carried out jointly with Gabor Domokos and Philip Holmes.

Real-valued valuations on Sobolev spaces

Dan Ma
Vienna University of Technology, Austria

Let $1 \leq p < n$. We are going to classify real-valued valuations on the Sobolev space $W^{1,p}(\mathbb{R}^n)$. For $p \leq q \leq \frac{np}{n-p}$, the qth power of L^q-norm turns out to be be the unique non-trivial continuous, $\mathrm{SL}(n)$ and translation invariant real-valued valuation on $W^{1,p}(\mathbb{R}^n)$ that is homogeneous of degree q (up to suitable scale). More generally, we also establish such kind of characterization without the assumption of homogeneity. The technique used in the proofs ralates closely to the centro-affine Hadwiger theorem on the space of convex polytopes.

Log-concavity properties of Minkowski valuations

Franz Schuster
Vienna University of Technology

Joint work with A. Berg, L. Parapatits, and M. Weberndorfer.

In this talk I will discuss log-concavity properties of homogeneous rigid motion compatible Minkowski valuations. These extend the classical Brunn–Minkowski inequalities for intrinsic volumes and generalize seminal results of Lutwak and others. Two different approaches which refine previously employed techniques are explored. It is shown that both lead to the same class of Minkowski valuations for which these inequalities hold.

On Brickell's theorem

Csaba Vincze
University of Debrecen, Hungary

Let K be a convex body in $\mathbb{R}^n$ and suppose that K contains the origin in its interior. Under some regularity conditions we can introduce a Riemannian metric g_{ij} as the Hessian matrix

$$g_{ij} = \frac{1}{2}\frac{\partial^2 L^2}{\partial y^i \partial y^j} = L\frac{\partial^2 L}{\partial y^i \partial y^j} + \frac{\partial L}{\partial y^i}\frac{\partial L}{\partial y^j}, \tag{1}$$

where L is the induced Minkowski functional. Brickell's theorem states that if K is symmetric about the origin and the space has zero curvature with respect to the metric g_{ij} then K must be an ellipsoid. This means that L comes from an inner product. By Schneider's trick the proof is based on Blaschke-Santaló's inequality.

In this talk we present a different version of Brickell's theorem by substituting the central symmetry condition with another one. To prove the main result we need the theory of Funk spaces. If the origin is moving in the interior of the body then K is working as a unit ball denoted by K_p for any point $p \in \mathrm{int}\, K$. The induced Minkowski functionals provide a metric space environment which is called the Funk space associated with K. The key observations are related to the asymptotic properties and the convexity of the function $r(p)$ measuring the volume of K_p, where the volume element $\sqrt{\det g_{ij}}$ is given by formula (1) using the Minkowski functional L_p point by point.

Covering the sphere by equal zones and related questions

Tamás Zarnócz
University of Szeged, Hungary

A zone of half-width w on the 2-sphere S^2 is the parallel domain of radius w of a great circle. László Fejes Tóth conjectured in 1972 that the minimum half-width of n equal zones that can cover S^2 is $\pi/(2n)$. This conjecture is solved only for $n \leq 4$. In this talk, we give a lower bound for the minimum width of n equal zones that can cover S^2. We will also consider the following related question: what is the maximum of the sum of the angles of n great circles on S^2?

This is joint work with F. Fodor and V. Vígh.