ON EXTENSIONS OF GRÜNBAUM'S INEQUALITY

Francisco Marín Sola

Universidad de Murcia (Spain)

(Partly joint work with Jesús Yepes Nicolás)

Given a compact set $K \subset \mathbb{R}^n$ of positive volume, if K is convex with centroid at the origin, then, a classical result by Grünbaum says that one can find a lower bound for the ratio $\operatorname{vol}(K^-)/\operatorname{vol}(K)$ depending only on the dimension of K, where K^- denotes the intersection of K with a halfspace bounded by a hyperplane passing through its centroid.

In this talk, among other results, we show that fixing the hyperplane H, one can find a sharp lower bound for the ratio $vol(K^-)/vol(K)$ depending on the concavity nature of the function that gives the volumes of cross-sections (parallel to H) of K. When K is convex, this inequality recovers the previous result by Grünbaum. To this respect, we also show that the log-concave case is the limit concavity assumption for such a generalization of Grünbaum's inequality.

Finally, we will give an alternative proof for the functional version of Grünbaum's inequality showing also how both the functional and geometric results are equivalent.