On discrete log-Brunn-Minkowski type inequalities

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The conjectured log-Brunn-Minkowski inequality says that the volume of centrally symmetric convex bodies $K, L \subset \mathbb{R}^n$ satisfies $\mathrm{vol}\big((1-\lambda)\cdot K +_0 \lambda\cdot L\big) \geqslant \mathrm{vol}(K)^{1-\lambda}\mathrm{vol}(L)^{\lambda}$, $\lambda \in (0,1)$, and is known to be true in the plane and for particular classes of n-dimensional symmetric convex bodies. In this talk we discuss some new discrete log-Brunn-Minkowski type inequalities for the lattice point enumerator $\mathrm{G_n}(\cdot) = |\cdot \cap \mathbb{Z}^n|$. Among others, we show that if $K, L \subset \mathbb{R}^n$ are unconditional convex bodies and $\lambda \in (0,1)$, then

$$\mathrm{G_n}\left((1-\lambda)\cdot\big(K+C_n\big) +_0 \lambda\cdot\big(L+C_n\big)+\left(-\frac{1}{2},\frac{1}{2}\right)^n\right) \geqslant \mathrm{G_n}(K)^{1-\lambda}\mathrm{G_n}(L)^{\lambda},$$

where $C_n = [-1/2, 1/2]^n$. Neither C_n nor $(-1/2, 1/2)^n$ can be removed. Furthermore, it implies the (volume) log-Brunn-Minkowski inequality for unconditional convex bodies. The corresponding results in the L_p setting for $0 < p < 1$ are also discussed.