L-Multialgebras and P-fuzzy Congruences

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Abstract

The purpose of this note is the study of L-multialgebras and fuzzy congruences of multialgebras. In this regards first the notion of a L-multialgebras are introduced and studied and then the notion of a P-fuzzy relations on multialgebra are given and is applied to introduce the notion of P-fuzzy congruence of multialgebras. Finally, the lattices of P-fuzzy (resp. strong) congruences of multialgebras is constructed and and it is shown that is complete.

Keywords: P-fuzzy set, P-fuzzy relation, L-multialgebra, compatibility, congruences [1]

1 Introduction

The concept of a hypergroup was introduced by F. Marty [22]. Since then many researchers studied in this field and developed; for example [10, 11, 27]. Several aspects of homomorphisms, subalgebras and subdirect decompositions of relational systems of multialgebras (hyperalgebra) developed in [23], [24] by Picket and in [16] by Hansoul. In [26] D. Schweigert studied the congruence of multialgebras. Ameri and Zahedi introduced the notion of hyperalgebraic systems [1].

As it is well known Zadeh in 1965 [28] introduced the notion of a fuzzy subset μ of a nonempty set X as a function from X to unite real interval $I = [0, 1]$. J.E. Goguen in [15] replace I by a complete lattice L in the definition of fuzzy sets and introduced the notion of L-fuzzy sets.

Rosenfeld defined the concept of a fuzzy subgroup of a group G [21]. and since then many researchers have worked in this area. Zahedi and others introduced and studied the fuzzy hyperalge-

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braic structures (for example [2, 3, 4, 12, 13, 25]). The purpose of this note is the study of L-multialgebras and fuzzy congruences of multialgebras. We introduce the notion of L-multialgebras, as a generalization of multialgebras and fuzzy algebraic systems investigate the basic properties of L-multialgebras. Then we introduce notion of P-relation and apply it to introducing the notions of P-fuzzy congruence of multialgebras. Then we give the basic results of these notions. In particular, we show that the set of all P-fuzzy congruences on a given multialgebra via natural order, forms a complete lattice.

2 Preliminaries

In this section we gather all definitions and simple properties we require of hyperstructures and fuzzy subsets and set the notions. In the sequel H is a fixed nonvoid set, $P^*(H)$ is the family of all nonvoid subsets of H, and for a positive integer n we denote for H^n the set of n-tuples over H (for more see [1]).

For a positive integer n a n-ary *hyperoperation* β on H is a function $\beta : H^n \rightarrow P^*(H)$. We say that n the *arity* of β. A subset S of H is *closed* under the n-ary hyperoperation β if $(x_1, \ldots, x_n) \in S^n$

implies that $\beta(x_1, \ldots, x_n) \subseteq S$. A nullary hyperoperation on H is just an element of $P^*(H)$; i.e. a nonvoid subset of H.

An n-ary *relation* on H is a subset of H^n. We also say that the arity of ρ is n. Orders and equivalence relations on H are the best examples of binary (i.e. 2-array) relations on H. Henceforth sometimes we use hyperoperation instead of the n-ary hyperoperation. A *hyperalgebraic system* $\langle H, (\beta_i, \mid i \in I), (\alpha_j \mid j \in J)\rangle$ is the set H with together a collection $(\beta_i, \mid i \in I)$ of hyperoperations on H and a collections $(\alpha_j \mid j \in J)$ of relations on H. $\langle H, (\beta_i, \mid i \in I)\rangle$ is called a *hyperoperational system* or a *multialgebra*. Notice the set $\langle H, (\alpha_j \mid j \in J)\rangle$ is called a *relational system*.

A subset S of a multialgebra $H = \langle H, (\beta_i, \mid i \in I)\rangle$ is a *submultialgebra* of H if for all $i \in I$, each hyperoperation β_i is closed on S, that is $\beta_i(a_1, ..., a_n) \subseteq S$, whenever $(a_1, ..., a_n) \in S^n$. The *type* of H is the map from I into the set N of nonnegative integers assigning to each $i \in I$ the arity of β_i.

Let ρ be an $h-ary$ relation on H. Extended ρ to $P^*(H)$ in two ways. Let $A_1, ..., A_h \in P^*(H)$ be arbitrary. Then

1) Set $(A_1, ..., A_2) \in \overline{\overline{\rho}}$ if $A_1 \times ... \times A_h \subseteq \rho$; i.e. if $(a_1, ..., a_h) \in \rho$ whenever $a_i \in A_i$ for all $i = 1, ..., h$.

2) For $h > 1$ set $(A_1\times, ..., \times A_h) \in \overline{\rho}$ if for every $1 \leq j \leq h$ and all

$a_i \in A_i$ for $i = 1, ..., h, i \neq j$ we have $(a_1, ..., a_h) \in \rho$ for some $a_j \in A$. For $h = 1$ (if $\rho \subseteq H$) set $\overline{\rho} = \overline{\overline{\rho}}(= P^*(\rho))$.

For example, let $h = 2$, let ρ be an equivalence relation on H and let $A_1, A_2 \subseteq H$. Then $(A_1, A_2) \subseteq \overline{\overline{\rho}}$ if and only if $A_1, A_2 \subseteq B$ for a block (also called equivalence class) of ρ, while $(A_1, A_2) \in \overline{\rho}$ if the set A_1 and A_2 meet exactly the same blocks of ρ; in other words, if both sets A_1 and A_2 have the same hull in ρ.

An h-ary relation ρ on H is *strongly compatible* with an n-ary hyperoperation β on H if either (i) $n > 0$ and for every $h \times n$ matrix $M = [m_{ij}]$ over H whose column vectors are all in ρ, the values of β on the rows of M form an $h-tuple$ in ρ; explicitly if $(m_{1j}, ..., m_{hj}) \in \rho$ for all $j = 1, ..., n$, implies

$$(\beta(m_{11}, ..., m_{1n}), ..., \beta(m_{h1}, ..., m_{hn})) \in \overline{\overline{\rho}} \qquad (1)$$

or

(ii) $n = 0$ and $(\beta, ..., \beta) \in \overline{\overline{\rho}}$ (where $\beta \in P^*(H)$ is the value of β). Strong compatiblity was introduced in [10]. If we replace $\overline{\overline{\rho}}$ by $\overline{\rho}$ we obtain the notion of compatiblity ([22] for equivalence relation and independently [1] for $h = 1, 2$).

A binary relation ρ on a set M is called *compatible* (resp. strong compatible) with an n-ary hyperoperation β if $x_1 \rho y_1, ..., x_n \rho y_n$ im-

plies that

$$\beta(x_1, ..., x_n)\overline{\rho}\beta(y1, ..., y_n),$$

$$(\beta(x_1, ..., x_n)\overline{\rho_S}\beta(y1, ..., y_n))$$

where for nonempty subsets A **and** B **of** M**,**

$$A\overline{\rho}B \Longleftrightarrow \quad (\forall a \in A \;\; \exists b \in B : a\rho b \;\; \textbf{and} \;\; \forall b \in B \;,\; \exists a \in A : b\rho a),$$

and

$$A\overline{\rho_S}B \Longleftrightarrow \quad \forall a \in A, \forall b \in B \;\; a\rho b.$$

Let $\langle H, (\beta_i, | \; i \in I) \rangle$ **be a multialgebra. A binary relation** ρ **on** M **is called (resp. strong)** *congruence* **if** ρ **is an equivalence relation and (resp. strongly) compatible with every** $\beta_i, i \in I$**.**

For $n > 0$ **we extend an n-ary hyperoperation** β **on** H **to** $P^*(H)$ **by setting for all** $A_1, ..., A_n \in P^*(H)$

$$\beta(A_1, ..., A_n) = \bigcup\{\beta(a_1, ..., a_h)|a_i \in A_i(i = 1, ..., n)\}.$$

Whenever possible we write a **instead of the the singleton** $\{a\}$**; e.g. for a binary hyperoperation** $\circ$ **and** $a, b, c \in H$ **we write** $a \circ (b \circ c)$ **for** $\{a\} \circ (\{b\} \circ \{c\}) = \cup\{a \circ u | u \in b \circ c\}$**.**

An equivalence relation on A **compatible (strongly compatible) with a multialgebra** H **on** A **is** *congruence* **(strong congruence) of**

H. Denote by $Con(H)(Cons(H))$ the set of all congruences (strong congruences) of H.

Let $H = \langle A, (\beta_i, \mid i \in I)\rangle$ be a multialgebra and let $\theta \in Con(H)$. Let $A'\{B_j | j \in J\}$ be the set of blocks of θ. For every $i \in I$ define β'_i on A' as follows:

Let $j_1, ..., j_{M_i} \in J$ be arbitrary and let $a_l \in B_{j_l}$ for $l = 1, ..., M_i$. Let

$$\beta'_i(B_{j_1}, ..., B_{j_{M_i}}) = \{B_j | j \in J, B_j \quad \textbf{meets} \quad \beta_i(a_1, ..., a_{M_i})\} \qquad (2)$$

Since $\theta \in Con(H)$, it can be verified that β'_i is well defined M_i-ary hyperoperation on A'. Call $H/\theta = \langle A', \{B_j | j \in J\}\rangle$ a *factor multialgebra* of H. If, moreover, $\theta \in Con(H)$, then every β'_i is singleton valued, i.e. an operation on A', and H/θ is an algebra. For semihypergroups this fact are in [10](see also [12], [19] and [20]), the general case is in [1].

We view binary relation on A as subsets of A^2 and so for a multialgebra H on A the sets $Con(H)$ and $Cons(H)$ are naturally ordered by set inclusion. First we characterize the poset $(Con(H, \subseteq)$. Recall that for a binary relations ρ and σ on A the relation product (also called*de Morgan product*) is

$$\rho \circ \sigma = \{(x, y) \in A^2 | (x, u) \in \rho, (u, y) \in \sigma \quad \textbf{for some} u \in A\}.$$

It is well known and easy to show that the relation product is associative with the unital element $\omega = \{(a,a)|a \in A\}$.

A *hypergroupoid* **is a multialgebra of type** (2), **that is a set** H **together with a (binary) hyperoperation** $\circ$. **A hypergroupoid** $(H, \circ)$, **which is associative, that is** $x \circ (y \circ z) = (x \circ y) \circ z$ **for all** $x, y, z \in H$ **is called a** *semihypergroup*. **A** *hypergroup* **is a semihypergroup such that for all** $x \in H$ **we have** $x \circ H = H = H \circ x$ **(called the** *reproduction axiom*).

Let H **be a hypergroup. A nonempty subset** K **of** H **is a** *subhypergroup* **of** H **if** $a \circ K = K = K \circ a$ **for all** $a \in K$.

An element e **in a hypergroup** $H = (H, \circ)$ **is called an** *identity* **of** H **if for all** $x \in H$

$$x \in (e \circ x) \cap (x \circ e).$$

A *polygroup* **is a semihypergroup** $H = (H, \circ)$ **with** $e \in H$ **such that for all** $x, y \in H$

$(i) e \circ x = x = x \circ e$;

(ii) **there exists a unique element,** $x^{-1} \in H$ **such that**

$e \in (x \circ x^{-1}) \cap (x^{-1} \circ x), x \in \bigcap_{z \in x \circ y} (z \circ y^{-1}), y \in \bigcap_{z \in x \circ y} (x^{-1} \circ z)$.

In fact a polygroup is a multialgebra of type $(2, 1, 0)$.

Let $H = (H, \circ)$ **be a polygroup. A subhypergroup** $K = (K, \circ)$ **of**

H **is a**

(i) *subpolygroup* **of** H**, in symbols** $K \leq_P H$**, if** $e \in K = K^{-1}$

(ii) *normal subpolygroup* **of** H**, in symbol** KH**, if for all** $x \in H$ **we get** $x \circ K = K \circ x$**.**

Denote by L **a complete distributive lattice. The meet, join, and partial ordering of** L **will be written as** $\wedge, \vee, \vee \leq$**, respectively. By an** *L-subset* **of** X**, we mean a function** μ **from** X **to** L**. The set of all L-subsets of** X **is called** *L-power* **subsets of** X **and is denoted by** L^X**. In particular, when** L **is** $I = [0, 1]$**, the L-subsets of** X **are called the** *fuzzy subset* **and the set** I^X **is refereed as the fuzzy power set of** X**.**

Let $\mu \in L^X$**. Then the set** $\{\mu(x)|x \in X\}$ **is called the** *image* **of** μ **and is denoted by** $\mu(X)$ **or** $Im(\mu)$**. The set** $\{x \in X|\mu(x) > 0\}$ **is called the** *support* **of** μ **and is denoted by** μ^* **or** $supp(\mu)$**.**

Let $\{\mu_i|i \in I\}$ **be a family of L-subsets of** X**, where** I **is a nonempty index set, then** $\bigcup_{i\in I} \mu_i$ **and** $\bigcap_{i\in I} \mu_i$ **are given by**

$$(\bigcup_{i\in I} \mu_i)(x) = \bigvee_{i\in I} \mu_i(x),$$

$$(\bigcap_{i\in I} \mu_i)(x) = \bigwedge_{i\in I} \mu_i(x).$$

Let $\mu \in L^X$**. For** $a \in L$**, define** μ_a **as follows:**

$$\mu^a = \{x \in X | \mu(x) \geq a\}.$$

μ^a **is called the** *$a-$level subset* **of** μ**.**

It is easy to verify that for any $\mu, \nu \in L^X$**,**

$(1)\mu \subseteq \nu, a \in L \Longrightarrow \mu^a \subseteq \nu^a,$

$(2)a \leq b, a, b \in L \Longrightarrow \mu^b \subseteq \mu^a$

$(3)\mu = \nu \Longleftrightarrow \mu^a = \nu^a \quad \forall a \in L.$

Definition 2.16. By an L_n *-relation* **of** X**, we mean a function** μ **from** X^n **to** L**.**

If $n = 2$ **we say L-relation instead** L_2**-relation.**

An L-relation R **of** X **is said to be an L-similarity relation if**

(i) **is reflexive, that is**

$$R(x, x) = 1, \quad \forall x \in X;$$

(ii) **is symmetric, that is**

$$R(x, y) = R(y, x), \quad \forall x, y \in X;$$

(iii) **is transitive, that is**

$$R(x, y) \geq \bigvee_{z \in X} R(x, z) \wedge R(z, y).$$

3 L-Multialgebras

In the sequel H denotes the multialgebra $H = \langle H, (\beta_i, \mid i \in I)\rangle$.

Definition 3.1. Let $H = \langle H, (\beta_i, \mid i \in I)\rangle$ be a multialgebra. We say that $\mu \in L^H$ is an *L-submultialgebra* of H, in symbol $\mu <_{LHA} H$, iff

(i) for every $i \in I$ such that arity n_i of β_i is positive, for all $a_1, ..., a_{n_i} \in H$ and all $z \in \beta_i(a_1, ..., a_{n_i})$

$$\mu(z) \geq \mu(a_1) \wedge ... \wedge \mu(a_{n_i}) \qquad (4)$$

in other words, every values of μ on the set $\beta_i(a_1, ..., a_n)$ is at least the least of $\mu(a_1), ..., \mu(a_n)$ and

(ii) for any nullary hyperoperation β and every $z \in \beta$

$$\mu(c) \geq \mu(x) \quad \forall x \in H.$$

Denote by $LHA(H)$, the set of all L-submultialgebras of H.

Examples 3.2. (1) Let $H = (H, \circ)$ be a hypergroupoid. Then $\mu \in L^H$ is a fuzzy subhypergroupoid of H if for all $x, y \in H$

$$\mu(z) \geq \mu(x) \wedge \mu(y) \qquad (5)$$

(2) Let $H = (H, \circ, e)$ be a hypergroupoid with an identity element e (considered as a nullary hyperoperation). Then $\mu <_{FHA} H$ if and only if μ satisfies in (3) and $\mu(e)$ is the greatest element of the range of μ. In this case we say that μ is an L-subhypergroupoid of H.

(3) Let$H = (H, \circ, ^{-1}, e)$ be a polygroup (considered as a multialgebra of type $(2, 1, 0)$). Then $\mu <_{FHA} H$ if and only μ satisfies (3) and $\mu(e)$ is the greatest element of the range of μ and $\mu(x^{-1}) = \mu(x)$ for all $x \in H$. Indeed from (2) we get $\mu(x^{-1}) \geq \mu(x)$ for all $x \in H$. From the unity of x^{-1} we get $(x^{-1})^{-1} = x$ and so $\mu(x) = \mu((x^{-1})^{-1}) \geq \mu(x^{-1})$ proving the equality. Then μ is an *L-subpolygroup* if $\forall x, y \in H$ the following conditions are satisfies:

$(i) \mu(z) \geq \mu(x) \wedge \mu(y), \forall z \in x \circ y;$

$(ii) \mu(x^{-1}) \geq \mu(x).$

Theorem 3.3 (First Representation Theorem. Let $\mu \in FS(H)$. If all nonempty t-level subset μ^t is a submultialgebra of H, then μ is an L-submultialgebra of H.

Theorem 3.4 (Second Representation Theorem). Let μ be a L-submultialgebra of H. Then every nonempty t-level subset μ^t is a submultialgebra of H.

Proof. Let β be an n-ary hyperoperation ($n \geq 1$) and $a = (a_1, ..., a_n) \in {\mu^t}_n$. Then for any $z \in \beta(a_1, ..., a_n)$

$$\mu(z) \geq \mu(a_1) \wedge ... \wedge \mu(a_n) \geq t,$$

that is $z \in \mu^t$, which means that μ^t is closed under β. For any nullary hyperoperation β and $z \in \beta$, by (ii) of Definition 3.1 $\mu(z) \geq \mu(x)$. Thus μ^t is closed under nullary hyperoperation, too. This complete the proof.

4 P-fuzzy Congruences

Let $\mu \in FS(A)$. Recall that the set

$$\mu^p = \{x \in A | \mu(x) \geq p\}$$

is the $p-$level subset or p-cut of μ for every $p \in L$.

The following, property known, proposition links P-fuzzy subset and this level sets.

Proposition 4.1. Let A be a nonvoid set and let and $P = (P, \leq)$ be a nonempty ordered set. A family $M = \{M^p | p \in P\}$ of subsets

of A is the family of all level subsets of P-fuzzy subset μ on A if and only if

(i) M **covers** A**,**

(ii) **for every** $a \in A$ **the set** $M_a = \{p \in P | a \in M^p\}$.

has a greatest element (i.e. there exists $g \in M_a$ **such that** $g \geq p, \forall p \in M_a)$.

Proposition 4.2. Let $h \in N_+$ and let $P = (P, \leq)$ proving $x \in \mu^p$ be a nonvoid ordered set. A P-fuzzy set on H^h is an h-ary P-fuzzy relation on A.

For a subset Q of P set $P = (P, \leq)$ be nonvoid order, let

$$Q^{\downarrow} = \{p \in P | p \leq q \quad \forall q \in Q\},$$

$$Q^{\uparrow} = \{p \in P | p \geq q \quad \forall q \in Q\}.$$

The set of the form $Q^{\downarrow}$ and $Q^{\uparrow}$ are the *Galois-closed sets* **on the left and right in Galois connection induced by $\leq$ on P. We write $Q^{\downarrow\uparrow}$ for $(Q^{\downarrow})^{\uparrow}$. The set $\{(Q^{\downarrow})^{\uparrow} | Q \subseteq P\}$, ordered by $\subseteq$, is the Mac Neil completion of $(P, \leq)$. If $(P, \leq)$ is $\wedge-$ semilattice then for every finite nonvoid subset $Q = \{g_1, ..., g_n\}$ of P we have $Q^{\downarrow\uparrow} = \{g_1 \wedge ... \wedge g_n\}^{\uparrow}$.**

Definition 4.3. Let $h \in N_+$ and μ be an $h - ary$ P-fuzzy relation on H. For an $h-$ary hyperoperation β on A is strongly compatible

with μ if for every $h\times n$ matrix on A with rows $r_1, ..., r_h$ and columns $c_1, ..., c_n$

$$\mu(\beta(r_1) \times .... \times \beta(r_n)) \subseteq \{\mu(c_1), ..., \mu(c_n)\}^{\downarrow\uparrow} \qquad (6)$$

where by $\mu(\beta(r_1) \times .... \times \beta(r_n))$ we mean $\bigwedge_{u_i\in\beta(r_i)} \bigvee_{i=1}^{n} \mu(u_i)$

Definition 4.4. Let $P = (P, \leq)$ be a nontrivial ordered set, let $h \in N_+$ and let μ be an h-ary P-fuzzy relation on A. For $n \in N_+$ are n-ary hyeroperation β on H is compatible with μ if for every $h \times n$ matrix with rows $r_1, ..., r_h$ and columns $c_1, ..., c_n$ for each $1 \leq i \leq h$, for every $1 \leq j \leq h, j \neq i$, for all $u_j \in \beta(r_j)$ there exists $u_i \in \beta(r_i)$ such that

$$\mu(u_1, ..., u_h) \in \{\mu(c_1), ..., \mu(c_n)\}^{\downarrow\uparrow} \qquad (7)$$

A nullary hyperoperation on H is compatible with μ if it is strongly compatible with μ.

A multialgebra $\langle H, (\beta_i, \mid i \in I)\rangle$ is compatible with μ if each β_i is compatible with μ.

Remark 4.5. If $P = (L, \vee, \wedge)$ is a complete lattice. Then the condition (7) **is expressible in lattice terms:**

$$\bigwedge_{i=1}^{h} \bigwedge_{j=1, j\neq i}^{h} \bigwedge_{u_j\in\beta(r_j)} \bigvee_{u_i\in\beta(r_i)} \mu(u_1, ..., u_h) \geq \bigwedge_{k=1}^{h} \mu(c_k) \qquad (8)$$

In particular, if $P = ([0,1], \leq)$ (the unit interval of real numbers with the natural order) then the P-fuzzy sets are the standard fuzzy sets and (8) becomes

$$min_{i=1,...,h} min_{j=1,...,h, j\neq i} inf_{u_j} \in \beta(r_j) sup_{u_i \in \beta(r_i)} \mu(u_1,...,u_h) \geq min_{k=1,...,h} \mu(c_k).$$

Proposition 4.6. Let $P = (P, \leq)$ be a nontrivial order, let $H = \langle H, (\beta_i, \mid i \in I) \rangle$ be a multialgebra and let μ be an h-ary P-fuzzy relation on H. Then μ is compatible (resp. strongly compatible) with H if and only if for every $l \in L$ the multialgebra H is compatible (strongly compatible) with the level relation μ^l.

Definition 4.7. Denote by F_{hPA}, the set of h-ary P-fuzzy relation on A. The set F_{hPH} is naturally ordered pointwise as a set of maps from H^h into the ordered set P: For $\mu, \nu \in F$ set $\mu \preceq \nu$ if and only if $\mu(a) \leq \nu(a)$ for all $a \in H^h$.

The following proposition expresses $\preceq$ in terms of the level relations.

Proposition 4.8. Let $P = (P, \leq)$ be an ordered set and $\mu, \nu \in F_{hPH}$. Thus $\mu \preceq \nu$ if and only if $\mu^P \subseteq \nu^p$ for all $p \in P$.

The following definition extends equivalence relations to P-fuzzy relations.

Definition 4.9. Let $P = (P, \leq)$ be an ordered set with the greatest

element 1. A binary P-fuzzy relation θ on H is a *similarity* if for all $x, y, z \in H$.

$$\theta(x,x) = 1, \theta(y,x) = \theta(x,y), \qquad (9)$$

$$\theta(x,z) \in \{\theta(x,y), \theta(y,z)\}^{\downarrow\uparrow}. \qquad (10)$$

Lemma 4.10. Let θ be a binary P-fuzzy relation on H such that $\theta(a,a) = 1$ for all $a \in H$. Then θ is a similarity relation if and only if all θ^p are equivalence relations.

Definition 4.11. Let H be a multialgebra and P an ordered set with a greatest element 1. A similarity θ on H is a P-fuzzy congruence (strong P-fuzzy congruence) with θ. We denote by $Con_P H$ ($Cons_P H$) the set of P-fuzzy congruences (strong P-fuzzy congruences) of H. Further we denote by ε the constant map (from A^2 into P) with value 1. If, moreover, H has a least element 0. We denote by η the map from A^2 into P defined by setting $\eta(a, a' = 1$ if $a = a'$ and $\eta(a, a') = 0$ if $a \neq a'$.

Lemma 4.12. Let P have a least element 0 and greatest element 1. Then ε is the least element and η the greatest element of $(Con_P H, \preceq)$ for every multialgebra H on A.

Proposition 4.13. If H is a multialgebra on A and P an ordered set with a least element 0 and greatest element 1. Then $(Con_P H, \preceq)$ is

a complete lattice with the least element η and greatest element ε.

Proof. If $P, Q \in Con_P H$, then $P \cap Q \in Con_P H$ and it is the greatest lower bound, while unique smallest P-fuzzy congruence containing $P \cup Q$, in fact it is the intersection of the family of all P-fuzzy congruence on H containing $P \cup Q$ is their least upper bound. It is easy to replacing the set $\{P, Q\}$ by an arbitrary family of P-fuzzy congruence, and so the lattice $(Con_P H, \subseteq, \cap, \vee)$ is a complete lattice. **Acknowledgment**

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