The Full Implies Strong Problem for Commutative Rings

Ross Willard (University of Waterloo)

In the theory of natural dualities it is still not known if every full duality is strong. I consider this question to be one of the most interesting and irritating problems of duality theory. It is known that for some finite algebras the answer to the question is yes for straightforward, "finitary" reasons. Davey, Haviar and Willard [2] gave the first example of an algebra (the three-element bounded lattice) for which the answer is yes but not for finitary reasons. This has been recently extended by Davey, Haviar and Niven [1] who answer the full-implies-strong question affirmatively for any finite bounded distributive lattice and show that the reason is not finitary whenever the lattice is not boolean.

In this lecture I continue the exploration of the question, this time in the domain of finite commutative rings.

Theorem. (1) If p is prime and $q = p^2$, then every alter ego for the ring Z_q which fully dualizes $\mathbf{ISP}(Z_q)$ also strongly dualizes $\mathbf{ISP}(Z_q)$. The reason is not finitary if $p \neq 2$.

(2) The same claim is true when Z_q is replaced by any finite indecomposable ring $R \in \mathbf{ISP}(Z_q)$. The reason is not finitary if $|R| \neq 4$.

This is joint work with Jennifer Hyndman and Todd Niven.

Reference

[1] B. A. Davey, M. Haviar, and T. Niven, When is a full duality strong?, manuscript, 2005.

[2] B. A. Davey, M. Haviar, and R. Willard, Full does not imply strong, does it?, *Algebra Universalis*, to appear.