On the subsemilattices of formular subalgebras and congruences of the lattices of subalgebras and congruences of universal algebras

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We study the questions of independence of subsemilattices of formular subalgebras (congruences) of universal algebras from the lattices of subalgebras (congruences) of these algebras.

The subalgebras $\mathcal{B} = \langle B; \sigma \rangle$ of the algebra $\mathcal{A} = \langle A; \sigma \rangle$ is open formular (parametric open formular), it there exists some quantifierfree formula $\phi(x)$ of the signature $\sigma(\phi(x, y_1, \ldots, y_n)$ and elements $b_1, \ldots, b_n \in A)$ such that $B = \{a \in A \mid \mathcal{A} \models \phi(a)\}$ $(B = \{a \in A \mid \mathcal{A} \models \phi(a, b_1, \ldots, b_n)\})$.

The down subsemilattice of all open formular (all parametric open formular) subalgebras of the lattice $Sub\ \mathcal{A}$ of all subalgebras of the algebra $\mathcal{A}$ we denote as $PSub\ \mathcal{A}$ $(PPSub\ \mathcal{A})$.

THEOREM 1. For any algebraic lattice L and its 0—1-down subsemilattices L_0, L_1 such that $L_0 \subseteq L_1$ there exists some universal algebra $\mathcal{A}$ and isomorphism φ of the lattice L on the lattice $Sub\ \mathcal{A}$ such that $\varphi(L_0) = PSub\ \mathcal{A}$, $\varphi(L_1) = PPSub\ \mathcal{A}$.

THEOREM 2. The equation $PSub\ \mathcal{A} = Sub\ \mathcal{A}$ is true for some finite algebra $\mathcal{A}$ iff all its inner isomorphisms (the isomorphisms between subalgebras of algebra $\mathcal{A}$) are its inner automorphisms (the automorphisms of subalgebras of algebra $\mathcal{A}$).

By analogy the definition of open formular subalgebra we define the concept of open formular congruences of the algebra $\mathcal{A}$. The down subsemilattice of the lattice $Con\ \mathcal{A}$ of all congruence of algebra $\mathcal{A}$ we denote as $Con\ OF\mathcal{A}$.

THEOREM 3. For any algebraic lattice L and 0—1-down subsemilattice L_0 of the lattice $0{+}L$ there exists some universal algebra $\mathcal{A}$ and the isomorphism φ of the lattice $0 + L$ on the lattice $Con\ \mathcal{A}$ such that $\varphi(L_0) = Con\ OF\mathcal{A}$.