Hereditary congruence lattices

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Inspired by a paper of John Snow we introduced the concept of a hereditary congruence lattice as follows. Let L be a sublattice of the lattice of all equivalence relations on a set A. We say that L is a hereditary congruence lattice, if for every complete 0–1 sublattice L' of L there exists a set of operations F' such that $L' = Con(A; F')$. By a result of Quackenbush and Wolk every finite distributive sublattice of $Eq(A)$ is a hereditary congruence lattice. John Snow proved that every finite lattice in the lattice variety generated by M_3 is the congruence lattice of a finite algebra, by constructing a hereditary congruence lattice representation of M_3^n. In a joint paper with Pál Hegedűs we generalized this to a much larger variety of modular lattices, again by constructing certain hereditary congruence lattices, in this case as sublattices of the congruence lattice of vector spaces over the 2-element field. It is an open question whether there is a hereditary congruence lattice isomorphic to M_4.