Non-deterministic Hypersubstitutions

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May 13, 2005

Abstract

A mapping σ which assigns to every n-ary operation symbol f an n-ary term of a given type τ is said to be a hypersubstitution of type τ. The concept of a hypersubstitution was introduced by Denecke, Lau, Pöschel and Schweigert in the 90th. Every hypersubstitution σ of type τ induces a mapping $\widehat{\sigma}$ on the set of all terms of type τ. Its application to a term t is that term which we get from t by replacing of each operation symbols f_i in t by the term $\sigma(f_i)$. This process will be more clear if we regard a term as a tree. Now we want to consider the case that one has more than one possibility for the replacement of any operation symbol in a given term t by only one application of a mapping to t. This can be realized by the concept of a non-deterministic hypersubstitution. Here the image of any operation symbol f_i is an element of a given set of terms instead of that for f_i determined term $\sigma(f_i)$. That means, the application of a non-deterministic hypersubstitution to a term gives a set of terms. We will introduce the theory of a non-deterministic hypersubstitution as well as of a non-deterministic solid (*nd*-solid) variety. Further, we give a relationship to the clone theory.