ON DISCRETE CONVEXITY AND ORDERED SETS

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Connections between order and convexity will be discussed.

On any ordered set there are several closure operators which exhibit different aspects of the behaviour of the convex hull operator in Euclidean space, such as the anti-exchange property or the separation of points by half-spaces. The latter is related to classical and more recent results, and to some open problems, concerning the representation of partial orders by linear orders.

Monotonicity properties of functions between ordered sets, of real-valued functions on the discrete hypercube in particular, can also be advantageously described from the perspective of convexity. These properties correspond to classes of functions that are themselves convex sets in function space, and closed under some additional conditions. Certain lattices of function properties can be characterized by a small number of closure criteria.