Rank is not rank!

Brian A. Davey (La Trobe University)

In 1997, Ross Willard [2] introduced a powerful but extremely technical condition related to strong dualisability in the theory of natural dualities. He gave a definition by transfinite induction of the **rank** of a finite algebra. The Oxford English Dictionary (OED) gives 12 different interpretations for the noun 'rank', including the one intended by Willard:

position in a numerically ordered series; the number specifying the position.

When I first saw the definition of rank, what immediately came to mind was one of the 15 OED definitions of the adjective 'rank', namely

having an offensively strong smell; rancid.

The aim of this talk is to show that I was completely wrong in my original assessment of the concept of rank. I will show that there is a very natural way to introduce the concept. Along the way, I shall give a characterization of those dualisable algebras that have rank 0, and show how rank 1 is related to both the injectivity of the algebra and to the congruence distributivity of the variety it generates.

The ranks of many finite algebras have been calculated. For example: the three-element Kleene algebra has rank 0, as does the two-element bounded distributive lattice (though its unbounded cousin has rank 1); for each prime p, the ring (with identity) of integers modulo p^2 has rank 1; every finite unar, and more generally every finite linear unary algebra, has rank at most 2; entropic graph algebras and entropic flat graph algebras have rank at most 2; every dualisable algebra that is not strongly dualisable has rank infinity.

There is still much to learn about the notion of rank—between rank 2 and rank infinity, no examples are presently known.

The results presented in this talk are part of the appendix on strong dualisability in a new text by Pitkethly and Davey [1].

References

[1] J. G. Pitkethly and B. A. Davey, *Dualisability: Unary Algebras and Beyond*, Springer, 2005.

[2] R. Willard, New tools for proving dualizability, *Dualities, Interpretability and Ordered Structures* (*Lisbon, 1997*) (J. Vaz de Carvalho and I. Ferreirim, eds), Centro de Álgebra da Universidade de Lisboa, 1999, pp. 69–74.