Semilattices of finite Moore families

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Common work with Gabriela Hauser Bordalo[1] and Bernard Monjardet[2]

Moore families (also called *closure systems*) are set representations for lattices. For instance, the Moore families called *convex geometries* represent lower locally distributive lattices (they are also in duality with the path-independent choice functions of the consumer theory in microeconomics). We present a review of a number of works on some sets of Moore families defined on a finite set S which, ordered by set inclusion, are semilattices or lattices. In particular, we study the lattice $\mathcal{M}_P$ (respectively, the semilattice $\mathcal{G}_P$) of all Moore families (respectively, convex geometries) having the same poset P of join-irreducible elements. For instance, we determine how one goes from a family $\mathcal{F}$ in these lattices (or semilattices) to another one covered by $\mathcal{F}$ and also the changes induced in the irreducible elements of $\mathcal{F}$. In the case of convex geometries, this allows us to get an algorithm computing all the elements of $\mathcal{G}_P$. At last, we characterize the posets P for which $|\mathcal{M}_P|$ or $|\mathcal{G}_P|$ is less than or equal to 2.

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