On the lattice of clones on three elements

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The talk is devoted to the lattice of all clones on three elements. The lattice of all clones on two elements was completely described by Emil Post. It turned out that all such clones are finitely generated and the lattice is countable. In 1959 it was proved that the lattice of all clones in three-valued logic has continuum cardinality. That is why, it seems hardly possible to obtain a complete description of the lattice and hence, it is interesting to find sublattices that can be completely described. In this talk we present two such results.

In 1955 Jablonskij described all maximal (also known as precomplete) classes in three-valued logic. Later, it was proved that all maximal classes except the class of all linear functions contain a continuum of subclasses. In spite of continuum cardinality we found a complete description of all clones of self-dual functions. This is the first maximal clone besides the clone of all linear functions to have such description.

All minimal clones on three elements were found by Csákány. For every minimal clone we found the cardinality of the set of all clones containing this minimal clone (the same problem was already solved for maximal clones). Moreover, for finite cases we completely describe the corresponding sublattices and for countable cases we discuss the possibility of obtaining such description.

In the end, we will discuss further perspectives in description of other sublattices of the lattice of all clones on three elements.

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