Minimal Clones and Maximal Centralizing Monoids

Hajime Machida
ICU, Japan

In the memorial paper of 1983, Béla Csákány determined all minimal clones on a three-element set. It still remains as an exciting source of further research.

B. Csákány, All minimal clones on the three element set, *Acta Cybernet.*, **6**, 1983, 227-238.

For a non-empty set A, $\mathcal{O}_A^{(n)}$ denotes the set of n-variable functions defined on A and $\mathcal{O}_A$ denotes the union of $\mathcal{O}_A^{(n)}$ for all $n > 0$. For a subset F of $\mathcal{O}_A$ the *centralizer* F^* of F is the set of functions in $\mathcal{O}_A$ which *commute* with all functions in F. A submonoid M of $\mathcal{O}_A^{(1)}$ is, by definition, a *centralizing monoid* if $M^{**} \cap \mathcal{O}_A^{(1)} = M$ is satisfied. In other words, M is a centralizing monoid if $M = F^* \cap \mathcal{O}_A^{(1)}$ for some $F \subseteq \mathcal{O}_A$. When $M = F^* \cap \mathcal{O}_A^{(1)}$ holds for a monoid M and a subset F of $\mathcal{O}_A$, we call F a *witness* of a centralizing monoid M. A *maximal* centralizing monoid is known to have a singleton witness whose component is a minimal function.

On a three-element set A, we have explicitly determined all centralizing monoids. Among them, maximal centralizing monoids draw particular attention. There are 10 maximal centralizing monoids. Surprisingly, we can restrict witnesses of all maximal centralizing monoids only to two kinds. Namely, 3 of maximal centralizing monoids have constant functions, which are minimal functions, as their witnesses and 7 of maximal centralizing monoids have majority minimal functions as their witnesses. In addition, we note that the results for constant function witnesses can be generalized to any finite set A with $k\,(> 2)$ elements.

This is a joint work with I. G. Rosenberg.

machida@math.hit-u.ac.jp