Lattice-valued identities and equational classes

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We investigate basic notions of universal algebra in the framework of lattice-valued structures. These are particular mappings which generalize the characteristic function, replacing the set $\{0, 1\}$ by a suitable complete lattice. In this way, starting with a classical algebra and a fixed lattice L, we obtain L-valued subalgebras (L-subalgebras for short), and the corresponding congruences, homomorphisms, products and other related notions. In addition, we replace the classical equality by a special compatible L-equivalence, so called L-equality. Next we define and investigate L-valued identities. These are formulas in which terms in the given language are related by an L-equality. Such an identity may be fulfilled by an L-subalgebra, while the underlying algebra need not satisfy the analogue classical identity. Among other properties, we prove that if an L-subalgebra of an algebra satisfies such an identity with respect to some L-equality, then there is a least L-equality such that the corresponding lattice-valued identity holds on the same L-subalgebra.

Next we introduce and investigate lattice-valued equational classes, for a given lattice L. These are defined with respect to a set of L-valued identities, and consist of lattice-valued algebras of the same type, fulfilling all given L-identities. Lattice-valued algebras are by definition L-subalgebras of classical algebras, equipped with an L-equality. In this lattice-valued framework we introduce basic notions of universal algebra: homomorphisms (H), subalgebras (S), and direct products (P). We prove that every lattice-valued equational class is closed under these three constructions (H, S and P), hence forming a lattice-valued variety.

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