Screens - a missing link between lattices and finite posets

Marcel Erné

Leibniz University Hannover, Germany

In the last years, a number of results about ideals and distributive laws in (bounded) lattices have been transferred by different authors to the setting of finite posets. A common realm for the validity of these results is provided by the notion of (upper and lower) screens: a lower screen is a poset in which the cone of all lower bounds of any finite subset is finitely generated (as a cut), and upper screens are defined dually. Screens may be regarded as "blurred lattices", but they are not the same thing as Benado's trellises! One of the basic reasons why so many facts extend from bounded lattices to screens is that the ideal lattice of a screen is isomorphic to the ideal lattice of the lattice of all finitely generated cuts. A major theorem says that the semiprime ideals of any lower screen are exactly the intersections of prime ideals (in ZF without choice, this is equivalent to the prime ideal theorem for Boolean algebras). Also, Nachbin's characterization of complementation in distributive lattices by the incomparability of their prime ideals extends to our setting: a lower screen satisfying suitable distributivity conditions at the top and at the bottom is complemented if and only if its prime ideals are exactly the maximal ideals.

erne@math.uni-hannover.de