

# Malcev families of quasivarieties closed under join or Malcev product

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We show that if  $K$  and  $L$  are quasivarieties of idempotent algebras satisfying  $\mathcal{P}$  where  $\mathcal{P}$  is any of the properties next listed, then the Malcev product of  $K$  and  $L$  satisfies  $\mathcal{P}$ , and therefore the variety generated by  $K \cup L$  satisfies  $\mathcal{P}$ . These properties are: “has a Taylor term”, “has a cube term”, “has meet-semi-distributive congruence lattices”, “has join-semi-distributive congruence lattices”, “has  $n$ -permuting congruences, for some integer  $n > 1$ ”.

On the other hand, we exhibit examples of finite idempotent algebras  $\mathbf{A}$  and  $\mathbf{B}$ , each of which generates a variety satisfying  $\mathcal{Q}$ , while  $\mathbf{A} \times \mathbf{B}$  does not, where  $\mathcal{Q}$  is any one of: “has a Malcev term”, “has Jónsson operations”, “has Day operations”.

These are joint results with Ralph Freese.

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