The Valeriote conjecture

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From the Galois correspondence between operations and relations it follows that the clone of term operations of any finite algebra $\mathbf{A}$ is equal to the clone of polymorphisms of a set of relations. If this set of relations can be chosen to be finite, the algebra $\mathbf{A}$ (or its clone) is called *finitely related*. Finitely related clones are nicer than general clones on a finite set (for instance, there is only countably many of them). On the other hand every clone can be "approximated" by them – each clone is an intersection of a descending chain of finitely related clones.

Recent developments have shown that there are many collapses of Maltsev conditions on the finite level, that is, certain inequivalent Maltsev conditions coincide for finite algebras. For instance, any finite algebra with an arbitrary Taylor term has a weak near unanimity term (Maróti, McKenzie), a Siggers term (Siggers), and a cyclic term (B., Kozik). Even more collapses occur for the finitely related algebras. An example of such a phenomenon is the fact that every finitely related algebra in a congruence distributive variety has a near unanimity term (conjectured by Zádori, proved by the author). A more general conjecture, the Valeriote conjecture (or the Edinburgh conjecture), predicted that every finitely related algebra in a congruence modular variety has "few subpowers" (it indeed generalizes the Zádori conjecture by a result of Marković and McKenzie). We give an affirmative answer.

The few subpowers property was discovered by Berman, Idziak, Marković, McKenzie, Valeriote and Willard in connection with the constraint satisfaction problem, where it captures the applicability of "Gaussian elimination like" methods for solving CSPs. An algebra $\mathbf{A}$ has *few subpowers* if the logarithm of the number of subalgebras of $\mathbf{A}^n$ is bounded from above by a polynomial in n (equivalently $\mathbf{A}$ has a cube term, or, equivalently, subpowers have generating sets of polynomial size).

In the talk I plan to discuss consequences of the Valeriote conjecture and give some ideas of the proof, which is based on cube term blockers (Marković, Maróti, McKenzie), ideas from a recent paper by Dmitriy Zhuk (The existence of a near-unanimity function is decidable, arXiv), directed Gumm terms (Kozik) and the absorption technique (which we are developing with Marcin Kozik).

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